$\langle x_1, y_1 \rangle$
ordered pairs
$\langle x_2, y_1 \rangle$
$\langle x_3, y_3 \rangle$
$\langle x_3, y_4 \rangle$

Q1) How many functions $X \to Y$

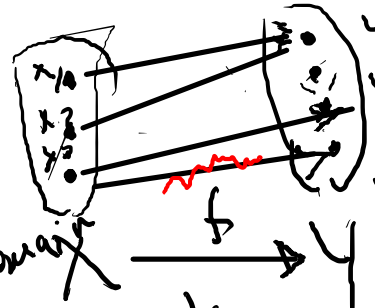
$|x,y| = |y,x|$
Combinatorics

$P_n = n! = n \times (n-1) \ldots \times 2$

3op. 2op. 1

1. length of words = 3

$m=3$ $n=4$



function $y = f(x)$

co-domain

ext diagr. 16×4

$4^n = 64$

3op. × 3op. × 3op.
1 2 3

$n=3$

ABC

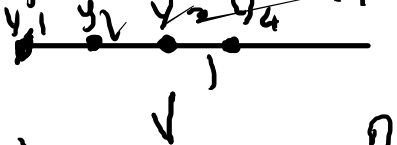
AAA, ABC

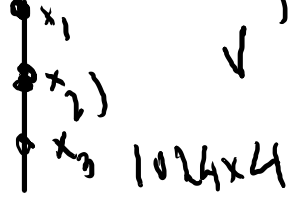
ACA

27

alphabets of symb.

Here n^m





Q2 How many pairs of form $\langle x, y \rangle$ where $x \in X$, $y \in Y$?

$4 \times 3 = 12$

2+2+2 | 3×4

$\langle x_2, y_2 \rangle$

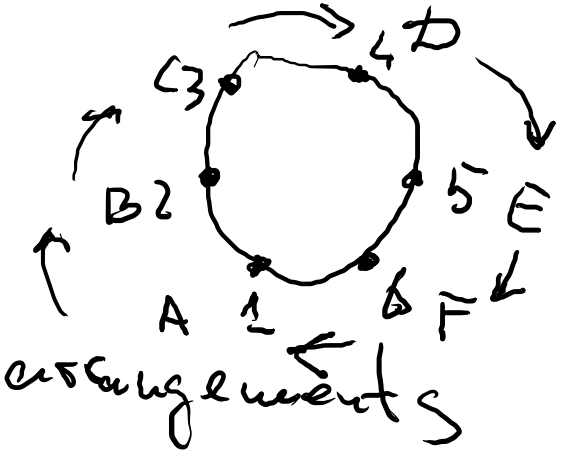
$2 \times 2 \times \cdots \times 2$ (12 times)

$2^{12} = 4096$

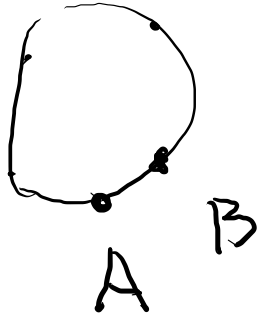
6 people

A B C D E F

round table



How many sitting arrangements



answer 1 : $6!$ (strict)

answer 2 : $\frac{6!}{6} = 5!$ (~~weak~~ flexible)

$1 \times 2 \times \dots \times 5 \times$ ~~6~~ =

How large should be a group of people to make it likely that two persons of the group have their birthdays on the same date?

$> \frac{1}{2}$ > 365

Let n be the number of people in the group

365 slots — 2 items → 365 options

$n \leq 365$

$n > 365$ | 365×365

S (total #) $= 365^n$

$S_{nr} = 365 \times 364 \times 363 \times$ ... $\times (365 - n + 1)$

Alice Bob | $\times 363 - n + 1$

no repeating birthdays

$\frac{365!}{(365-n)!}$ //

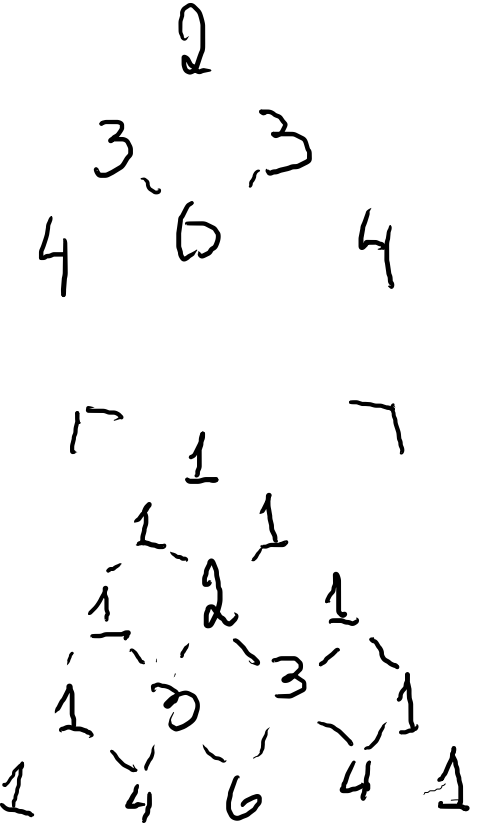
$C(n) = \frac{365!}{(365-n)! \, 365^n}$

$n > 365$

$\frac{S_{nr}}{S} < \frac{1}{2}$

$> \frac{1}{2}$

Pascal Triangle



Newton Binom

$$(a+b)^2 = a^2 + 2ab + b^2$$

polynom

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

monom

$$(a+b)^4 = (a+b)^3(a+b) = a^4 + 3a^3b + 3a^2b^2 + ab^3$$
$$a^3b + 3a^2b^2 + 3ab^3 + b^4$$
$$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$\vdots$$

$$(a+b)^n$$

$$a^n + ?a^{n-1}b + ?a^{n-2}b^2 + \cdots + ?a^{n-k}b^k + \cdots + ?ab^{n-1} + b^n$$

Numeration systems | Syntax for numbers

2021

α' ιβ' κα'

2×10² + 0×10 + 2×10 + 1

α' β' γ' δ' ε'

1 3 4

μ ν ο π ρ

2, 5 7 5+7=12

101 4×20 1 | 0, 1, 2, 3, 4 / 5, 6, 7, 8, 9 | 102 / 12

10 60 20 60+10 70 | 127 | 1 / 10

√2

1/3

1965

M CM LXV

MMXX 2000

I - 1

II + II = 2

III - 3

IIII (IV) 4

V = 5

VI

X 10

IX 9

L 50

C 100

D - 500

M - 1000

$$\underbrace{(a+b)\times(a+b)\times\ldots\times(a+b)}_{n \text{ times}} = \left| \begin{array}{l} \overbrace{a\times a\times}^{k \text{ times}} \quad \overbrace{\quad}^{(n-k) \text{ times}} \\ n \text{ choices} \end{array} \right.$$

$n=10$
$k=3$

$= \left| \begin{array}{l} \text{different arrangements} \\ \text{but} \\ \text{the same combination!} \end{array} \right.$

$k<n$

$C_n^k \; a^{n-k} \; b^k = a^k b^{n-k}$

$\| \; C_n^{n-k}$

$A_n^k = \frac{n!}{(n-k)!}$

$C_n^k = \frac{n!}{(n-k)!\,k!}$

Newton Th.

$$(a+b)^n = a^n + C_n^1 a^{n-1} b + C_n^2 a^{n-2} b^2 + \dots$$

$$\dots + C_n^k a^{n-k} b^k + \dots + C_n^{n-k} a^k b^{n-k} + \dots C_n^1 a b^{n-1} + b^n$$

$$\frac{n!}{(n-k)k!} = C_n^k = C_n^{n-k} = \frac{n!}{k!(n-k)!}$$

$n-(n-k)=k$