Arithmetic
(Number Theory)

$\mathbb{N}$ 1. natural numbers: 1, 2, 3, ... etc.
(et cetera)
?

[Definition]

definiendum := definiens
(ex. circle) — bla-bla....

$[m = n+1]$

no n s.t. $S(n) \to (n+1)$ $\quad S(n) \to n$

$S(n) = 1$ $\quad$ axs 1-3 $\to 1, S(1), S(S(1)) \ldots$

$S(n) \geq n$

ax.4 if $S(n) = S(m)$ then $n = m$ $\quad$ counterposition

$\Updownarrow$

if $n \neq m$ then $S(n) \neq S(m)$

$f(x) = x^2$

$f(1) = 1$

$f(-1) \neq$

$\mathbb{N}$: 1, 2, 3, ...

satisfaction

standard model

isomorphism

$1 < 2 < 3 < \dots < n < \dots$

$2 < 3 < 4 < \dots < n+1 < \dots$

non-standard models

Exercise: Prove in PA that

$$3 \neq 5$$

Where

$3 := S(S(1))$

$5 := S(S(3)) = S(S(S(S(1))))$

Hint: use "reductio ad absurdum"

if $S(m) = S(n)$ then $m = n$ ax.4

Let $\overline{3 = 5}$

$S(S(1)) = S(S(3))$

$S(1) = S(3)$

$S(1) = S(S(S(1)))$

$1 = S(S(1))$

contr. ax.3 //

$S(n) = n + 1$

Ex. 2.

for no n $\quad n = S(n)$ $\quad \boxed{n \neq n+1 \text{ for all } n}$

$\Downarrow$ equivalent

$S(n) = S(S(n))$

$p \to p$

[Induction by n

a) base case $n = 1$

$1 = S(1)$ false by ax. 3

b) Ind. hyp. $k = S(k)$ is false

Ind. step. $(k = S(k)) \to [S(k) = S(S(k))]$

$\boxed{k = S(k)}$

$\Big\downarrow$ log. eq. by ax. 3

$\Big\downarrow$

$\boxed{S(k) = S(S(k))}$ ∎

$(11,13)$ $(17,19)$ $(29,31)$ ... $2+2$

$2+S(1)=S(2+1)$!

a) $n+1 := S(n)$

b) $n+S(m) = S(n+m)$

a) $2+1 := S(2) := S(S(1))$

~~$3+1 = S(3) = S(S(2))$~~

$2+2 = S(S(S(1))) = 4$

$1+1 := S(1) := 2$

$2+2 = 4$

$2+$

80 80 10

$\frac{1}{2} = \frac{2}{4} = \frac{6}{12}$

Integers
[Whole number.]

$(-1)\times(-1) = 1$

$\mathbb{N} \times \mathbb{N} \to \mathbb{N}$

$2+x=5$

$5+x=2$

ordered pairs

Genetic introduction

[not axiomatic]

$\langle 1,1 \rangle$
$\langle 2,2 \rangle$
$\vdots$
$\langle n,n \rangle$
$\vdots$

$n-n=0$ for all $\mathbb{N}$

$\langle 1,1 \rangle \sim \langle 2,2 \rangle \sim \langle 3,3 \rangle \cdots := 0$

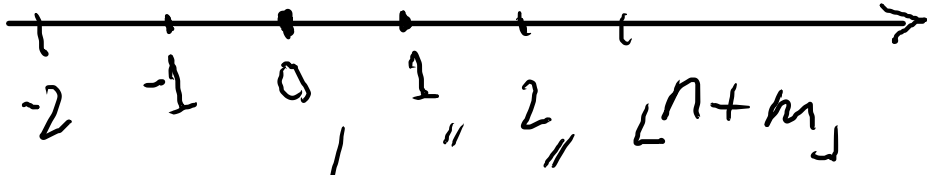
0 0.0

$\frac{1-1}{3-3}$

$\frac{}{4-3}$

$=$

$[m-n := m+(-n) \quad x=-3]$



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What is zero?

$3-2$
$\overline{3-3}$

$4-3$ $2+x=2$

$\overline{3-4}$

$\mathbb{Z}$ $\exists 1 := \langle 2, 1 \rangle \sim \langle 26, 25 \rangle \ldots$ $(\mathbb{N})$

$-1 := \langle 1, 2 \rangle \sim \langle 25, 26 \rangle \sim \ldots$

$m, n, m', n' \in \mathbb{N}$

$\langle m, n \rangle \sim \langle m', n' \rangle$

$m - n = m' - n'$

$m + n' = m' + n$

$[m, n] \in \mathbb{N}$

$m - n = -1$

$1 - 2$

$\frac{m}{n} \sim \frac{m'}{n'}$

$m \times n' = n \times m'$

$m, n, k, l \in \mathbb{N}$

$z_1, z_2 \in \mathbb{Z}$

$z_1 = \langle m, n \rangle$

$z_2 \langle k, l \rangle$

$z_1 + z_2 := \langle m+k, n+l \rangle$

$(-1) + (-2) = \langle 1+3, 2+5 \rangle =$

$= \langle 4, 7 \rangle = -3$

$-1 = \langle 1, 2 \rangle$

$-2 = \langle 3, 5 \rangle$

$Z_1 \times Z_2$

$Z_1 = \langle m, n \rangle$

$Z_2 = \langle k, l \rangle$

$(m-n)(k-l) = mk - ml - nk + nl$

Homework explain why -1

$Z_1 = -1$

$Z_2 = -2$

$(-1) \times (-2) = 2$

$Z_1 = \langle 1, 2 \rangle$

$Z_2 = \langle 3, 5 \rangle$

$\langle 3, 10 \rangle = -7$

$\langle mk+nl, ml+nk \rangle$

$\langle 3+10, 5+6 \rangle = \langle 13, 11 \rangle$

$13 - 11 = 2$