$z \in \mathbb{Z}$ $\quad m, n \in \mathbb{N}$

$z := \langle m, n \rangle$

$z' := \langle m', n' \rangle$

$p = z \times z'$

$(m-n)(m'-n') =$

$= mm' - mn' - m'n + nn'$

$p = \langle mm' + nn', \; mn' + m'n \rangle$

$z = m - n$

if $m < n$

then z is negative

$- \times - = +$

if $m < n$

$m' < n'$, then

$\langle mm' + nn', \; mn' + m'n \rangle$

$[mm' + nn' > mn' + m'n$ when $[m<n$ and $m'<n']$

$\Uparrow$

$nn' - m'n > mn' - mn'$ true

$n(n' - m') > m(n' - m')$ true

$\Uparrow$

$n > m$ true

$a + (-a) = 0$

Proposition 1

opposit to given int. a, denote $(-a)$, Suppose that $(-a)'$ is opp to a
is unique! Proof □

Proof: by uniqueness

Lemma. $-(-a) = a$

$(-a)' + (a + (-a)) = 0 + (-a) = (-a) =$

$= (-a)' + 0 = (-a)'$ =

Let $-(-a) = b$

$b + (-a) = 0$

if $b = a$, then

b is opp

$a, b > 0$

$ab = ba$

$(-a) \times (-b) = ab$

Ex. $-a = a(-1)$

$-a = (-1)a$

$(-1)a \times (-1)b =$

$= ab \times (-1) \times (-1)$

$(-1) \times (-1) = 1$

$a + x = b$ ~~$b > a$~~ [Rational numbers] $\mathbb{Q}$

$x = b + (-a)$ $a, b, x \in \mathbb{Z}$

$b - a$ in $\mathbb{Z}$

for all a | $a + 0 = 0 + a = a$

for all a there ex. | $a + (-a) = 0$

$a \cdot x = b$ $a, b, x \in \mathbb{Q}$

$x = \frac{b}{a}$ in $\mathbb{Q}$ when $a \neq 0$!

what happens when $a = 0$

$0 \cdot x = b$, x any rational $b = 0$

$b \neq 0$

$0 \cdot x = 2$ [no such x exists.]

Th. $0 \cdot b = (a + (-a)) \cdot b =$ expand

$= a \cdot b + (-a) \cdot b =$ field

$= a \cdot b + (-(a \cdot b)) = 0$ Q

a

$a \neq 0$ $x = b/a$

$a = 0$

$b \neq 0$

$b = 0$ - any rat. is a root

$\frac{1}{2}, \frac{2}{3}$ $\quad \frac{m}{n} + \frac{m'}{n'}$ $\quad m \in \mathbb{Z}$

$n \in \mathbb{N}$ (w/out 0)

$\langle m, n \rangle \quad \langle m', n' \rangle$

$\frac{m}{n} \times \frac{m'}{n'} = \frac{mm'}{nn'} \quad \langle mm', nn' \rangle$

$\frac{2}{3} + \frac{3}{4} = \frac{2\times4 + 3\times3}{3\times4} \quad \frac{m}{n} + \frac{m'}{n'} = \langle mn' + m'n, nn' \rangle$

$ax = b$, $a, b \in \mathbb{Q}$

eq. of degree 1.

$2x = 3$ - linear eq.

$a \neq 0$

$2x^2 = 3$

$ax^2 + bx + c = 0$

$x^2 + \frac{b}{a} + \frac{c}{a} = 0$

$x^3 + \ldots = 0$ eq. deg. 3

$x^4 + \ldots = 0$

for all $r_1, r_2 \in \mathbb{Q}$ $\quad r_1 < r < r_2$ (transcendental)

$\frac{r_2 - r_1}{2}$ $\quad (\pi)$

e // numbers

overkill

$\lfloor x^2 = 2 \rfloor$ $\quad$ no $r = \frac{m}{n}$ s.t. $\left(\frac{m}{n}\right)^2 = 2$

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

algebraic

nat. $\quad$ int. $\quad$ rat. $\quad$ A - (reals)

$x_1 = \sqrt{2}$

$x_2 = -\sqrt{2}$ $\quad m^2 = 2n^2$

no $m, n \in \mathbb{N}$ s.t.

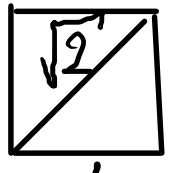
$\lfloor 1{,}4142 \rfloor \frac{1}{2}$ $\quad \frac{m}{n}$

-1

$0 \quad 1 \quad \sqrt{2} \quad 2 \quad r_1 \quad r_2$

side

$\lfloor \text{length} \rfloor$



1

$1 < r < 2$

? $\left(\sqrt{2}\right)^2 = 2$

Systeme

$\sqrt{2}$ $\sqrt{3}$ $a, b \in \mathbb{Q}$

$(a + \sqrt{2}\, b)$

$\pi_4 < \pi = \pi_v$

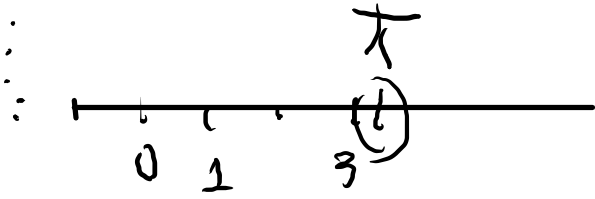
$\pi = 3,1415926\ldots$

$\pi_0 = 3/4$

$\pi_1 = 3,1$

$\pi_2 = 3,14$

$\vdots$



$C_\pi = \pi_0, \pi_1, \ldots, \pi_k \ldots$

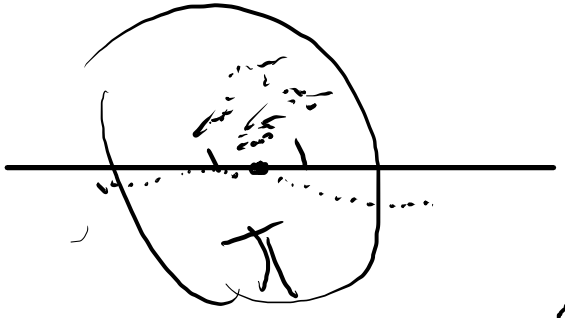
$\lim C_\pi = \pi$

$\langle m, n \rangle \sim \langle m', n' \rangle$

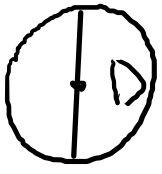
$m - n = m' - n'$ (b) rational numbers

Cauchy series

Dedekind Cuts



ℓ (circumference)



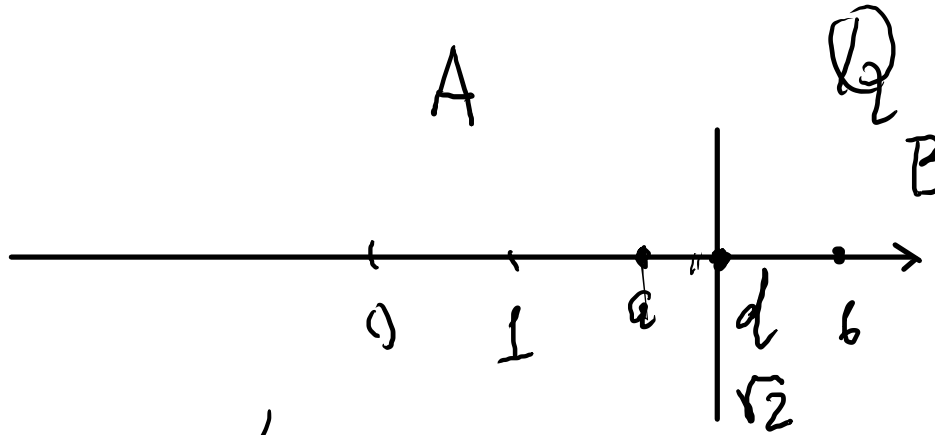
$\pi = \frac{\ell}{D}$

$D/2 = R, \quad \ell = 2\pi R$

Cut

$\alpha \in \mathbb{R}$

partition $\{A, B\}$ of $\mathbb{Q}$



$A \cap B = \emptyset$

$A \cup B = \mathbb{Q}$

if $a \in A$ and $b \in B$ then $a < b$

if $a \in A$ and $c < a$ then $c \in A$

if $b \in B$ and $c > b$ then $c \in B$

B has no minimal element

complex numbers

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$$

$$x^2 = -1, \quad x \in \mathbb{R}$$

no root

$$x^5 = 1$$

$\mathbb{C}$