

$E \notin N$

Cardinality / Mächtigkeit

even numbers

\cup

$N : 1, 2, 3, \dots, k, \dots, m/2, \dots$ to generate one-to-one corr.

$E : 2, 4, 6, \dots, 2k, \dots, m, \dots$ one-to-one corr.

$k \in N$

$\|N \simeq E\|$
 \Downarrow "size" of set M

$$\begin{array}{ccc} N & \xrightarrow{x_2} & E \\ \Downarrow & \swarrow 1/2 & \Downarrow \\ k & \rightarrow & 2k \end{array}$$

$M \simeq N$ - "the same size" (perhaps rougher)
 $(N \ni m/2 \leftarrow m \in E)$
 $M \subset N$

Def $M \simeq N$ iff there exists one-to-one corr. betw. M and N

$$n!$$

a b c d e
1 2 3 4 5

$$5! = 1 \times 2 \times 3 \times 4 \times 5 = \\ = 120$$

$$M = \{a, b, c, d, e\}$$

$$M \neq \emptyset$$

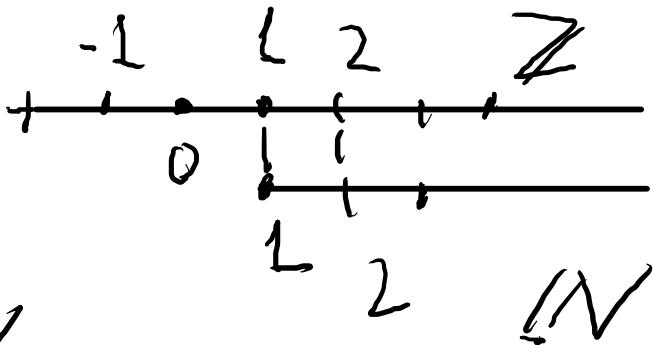
$$N = \{a, b\}$$

proper subset

$$N \neq M$$

$$2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$$

$$\emptyset \subseteq M$$
$$M \subseteq M$$



[Def.] $\|M\| > \|N\|$

iff: ① ex. $M' \subseteq M$, T. 2. $M' \cong N$, no
 ② ex. $N' \subset N$ T. 2. $M \cong N'$
 ③ $M \cong N$

Th. Bcc un. & s. cplnacev no množstvom:

jake množstvo množ. M, N besivo očiso (u členovs opis)
 už e.g. $3x$ yftb.

$$\begin{array}{l} 1. M \cong N \\ 2. \|M\| > \|N\| \\ 3. \|N\| > \|M\| \end{array} \vdots$$

$$\underline{\text{?}} \quad ? \quad \mathbb{Z} \supseteq \mathbb{Z}^+ / \mathbb{Z} \cong \mathbb{N}$$

C2elTne / Countable

$0, 1, -1, 2, -2, \dots$

$$\frac{m}{n}$$

$$k = 2, 3, \dots q = \frac{m}{n} \in \mathbb{Q}, \quad \mathbb{Q} \subset \mathbb{N}$$



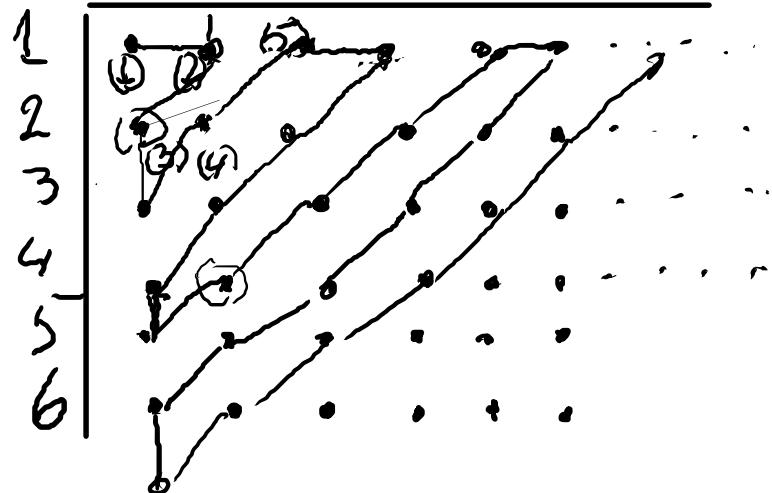
\dots

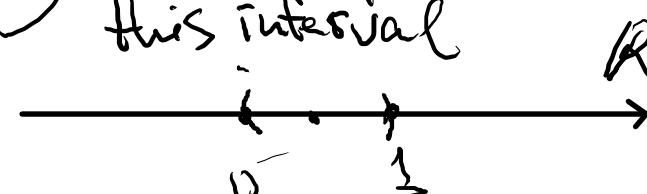


(rationals)
 \mathbb{Q} is countable

$$\frac{\sqrt{5}}{2} = \frac{2}{4} = \frac{5}{10}$$

1 2 3 4 5 6 \dots



- Generalized Cantor Th
 for any set M
 $\|P(M)\| > \|M\|$
 $\|N\|$
 $\mathbb{Q} > \|N\|$
 "the same size"
 "equivalent"
 $M \sim N$
 diagonal proof
- Examples of non-countable sets
 Number reductio incorrecte.
- (1) $[0, 1] \subseteq \mathbb{R}$ (1) set of points within this interval
 
 - (2) set of all 0-1-Seqs. binary fraction.
 
 - (3) $P(N)$ set of all subsets of set N
 - $\in \mathbb{Q}$ 1 2 3 4 5
 - $P \subseteq N$

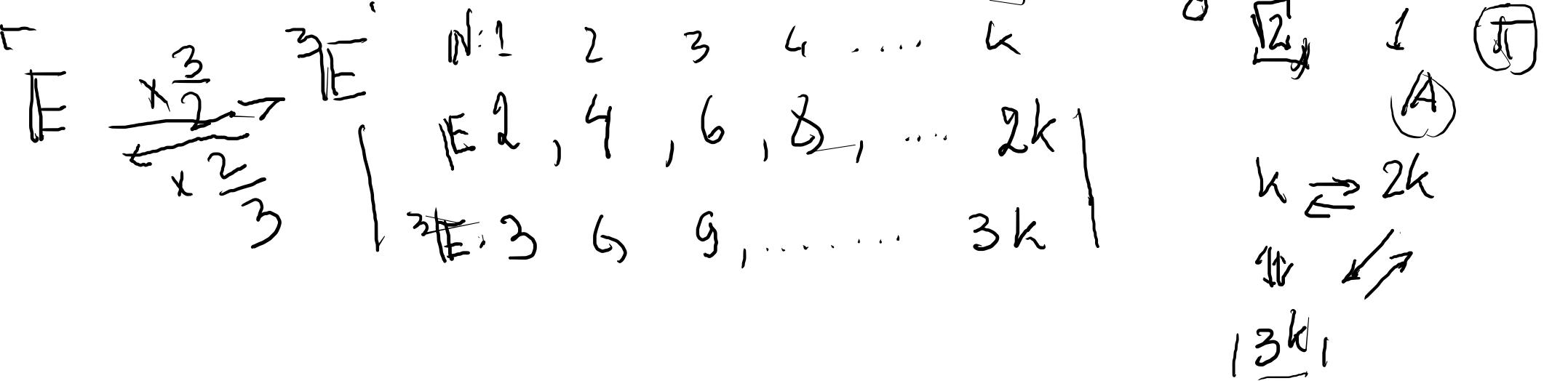
[Continuum Hypothesis]

to continue

continuous (transf.)

$$\text{||} \mathbb{N} \text{||} < ? < \text{||} \mathbb{R} \text{||}_{\#}$$

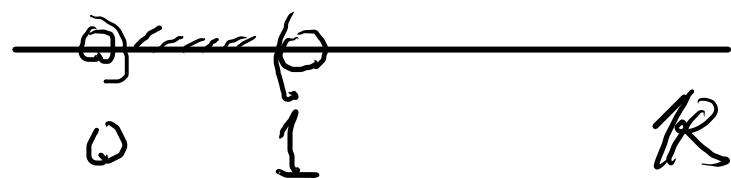
Georg Cantor: NO(?)



$$\|P(\{0,1\})\| = \underline{4}$$

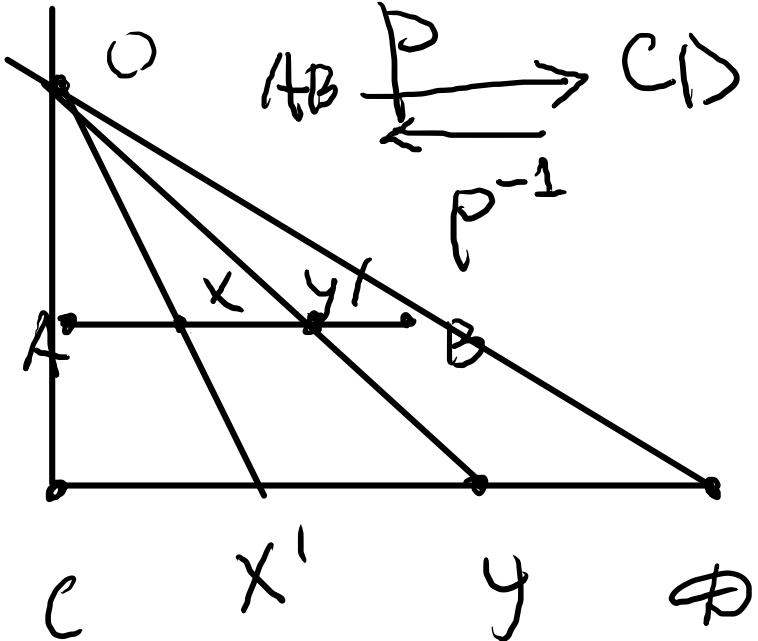
?
 $\{a, b\}$

Prop. inf. sets are larger
than finite sets!



Sets of points
↓

$AB \underset{?}{\sim} CD$
yes!



$$\frac{x \xrightarrow{P} x'}{P(x) = x'} \quad x \in AB$$

$$y \in CD$$

$$y \xrightarrow{P^{-1}} y'$$

PAG

(sets)

$$A \xrightarrow{f} B$$

domain \mathcal{A}

$$B = \{t, f\}$$



$$R \times R \xrightarrow{\text{arguments}} B$$

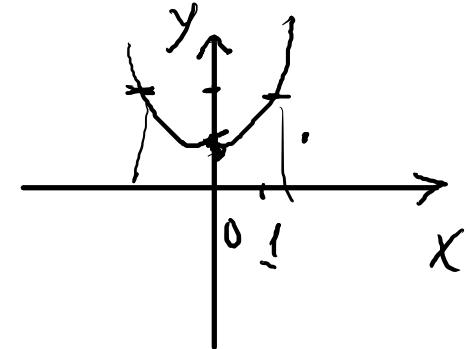
$$\begin{matrix} a \\ f(a) \end{matrix} \xrightarrow{f} b$$

arguments

Functions and maps.

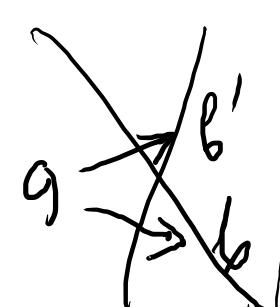
to map
(or function)

$$f: A \rightarrow B$$



$$R \rightarrow R \quad Q \rightarrow Q$$

$$y = f(x) = x^2 + 1 \quad N \rightarrow Q$$



$$f(a)$$

$$x=2$$

$$N \rightarrow E$$

$$f(n) = 2n$$

$$Q \rightarrow Q$$

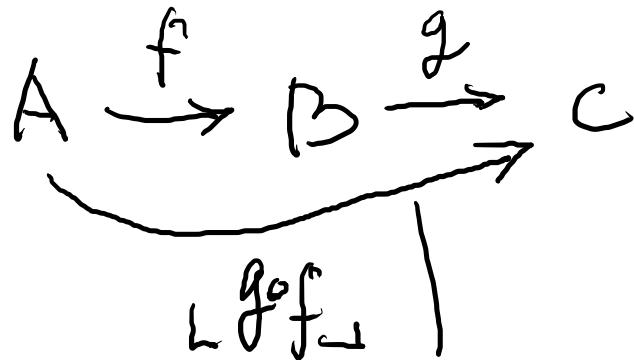
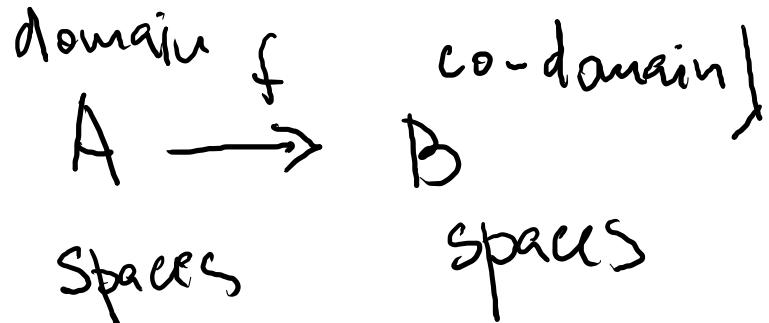
$$f(n) = \frac{n}{2} - 1$$

$$x=2$$

$$f(2) = 5$$

$$a \times b = b \times a$$

// Composition of functions (maps) //



|| inverse function.

$A \xrightarrow{f: x^2} B \xleftarrow{f^{-1}: \sqrt{x}}$

$f: \mathbb{R} \rightarrow \mathbb{R} \rightarrow \mathbb{R}$

$f(x) = x^2$

$g \circ f = f \circ g$
non-constant

$$f(y) = y + 1$$

$$\begin{aligned} & \downarrow \\ & g(f(x)) = \\ & = g(x) = \\ & = x^2 + 1 \end{aligned}$$

so far we have
 $(x+1)^2 =$
 $= x^2 + 1$