place-holders for relata

Equivalence (relation) $\_ \sim \_$

1) reflexivity $x \sim x$ (for all x, y, z)

2) symmetry

if $x \sim y$ then $y \sim x$

3) transitivity

if $x \sim y$ and $y \sim z$ then $x \sim z$

Theorem:

Eq. relation on set M induces a partitioning of M into eq. classes $C_1, \ldots, C_k$ s.t.

$$\bigcup_{i=1}^{k} C_i = M, \qquad C_i \cap C_j = \emptyset \text{ when } i \neq j.$$

Proof. Take $x \in M$ and all y s.t. $x \sim y$

$C_i = [x] = \{y \mid x \sim y\}$

1) $x \in [x]$ (refl.)
2) if $x \sim y$ then $[x] = [y]$
3) Suppose (symm.) $[x] \cap [y] = D \ni z$

then $z \sim x$ and $z \sim y$

(symm.) $x \sim z$ and $z \sim y$, then (trans.) $x \sim y$ $[x] = [y]$

Partial Order $\leq$

1) reflexive
2) anti-symmetric

if $x \leq y$ and $y \leq x$ then $x = y$

3) transitive

$\leq$

$M \subseteq P$ and $P \subseteq M$ then $M = P$

X y

Ex. 1 $m, n \in \mathbb{N}$ $\quad mRn \overset{\text{def.}}{\iff} (m,n) > 1$

common divisor

Check: 1. refl.

$mRm \iff (m,m) > 1$

$(6,8) = 2$

$6R8$

take $m = 1$ $\quad (1,1)$

$(6,5) = 1$

$6 \not R 5$

there is no common div. > 1

Hence R is not reflexive

$\{k, -k\}$ for all $k \in \mathbb{N}$

$\{0\}$

Ex. 2 $k, m, n \in \mathbb{Z}$ (integers)

It is an equivalence!

$mRn \iff m^2 = n^2$

$m = 2$

$n = -2$

1) refl. mRm $m^2 = m^2$ true!

$2R(-2)$

2) symm

if mRn then nRm | if $m^2 = n^2$ then $n^2 = m^2$ true!

$m = 2$

$n = 3$

$4 \neq 9$

$2 \not R 3$

3) trans. if kRm and mRn then kRn | if $k^2 = m^2$ and $m^2 = n^2$ then $k^2 = n^2$ true!

$m, n \in \mathbb{Z}$

$m^2 \leq n^2$?

if $m^2 \leq n^2$ and $n^2 \leq m^2$ then $m = n$ false!

not a p.o.

(Algebraic)

set | Group theory | Structure

$\langle G, \otimes \rangle$ binary operation

poset

result (output)

$g_1, g_2, g_3 \in G$ | $2+3=5$ $\mathbb{N}$

operanda (summands)

1) $(g_1 \otimes g_2) \otimes g_3 =$

$= g_1 \otimes (g_2 \otimes g_3)$ (associativity)

2) there ex. $e \in G$ (unit)

s.t. for all $g \in G$ $e \otimes g = g \otimes e = g$

3) for all $g \in G$ there ex. g^{-1} s.t. $g \otimes g^{-1} = g^{-1} \otimes g = e$

(inverse)

$\langle \mathbb{Z}, + \rangle$ | additive group of integers | $z \in \mathbb{Z}$ forall z | $e \overset{?}{\in} \mathbb{Z}$

$z + e = e + z = z$

$e = 0$!

v 1/ assoc

v 2/ unit

3/

$z + z^{-1} = 0$

-2

$z + (-1) \cdot z = 0$ | $z = 3$ | $3 + x = 0$?

$3 + (-3) = 0$

$\langle \mathbb{N}, + \rangle$ is it a group?

"concatenation"

$\underbrace{abc}_{w_1} \otimes \underbrace{xy}_{w_2}$

$abcxy$

monoid

Further examples of groups

$e = \begin{pmatrix} a & b & c \\ a & b & c \end{pmatrix}$

1) permutations.

$p_1 = \begin{pmatrix} a & b & c \\ b & c & a \end{pmatrix}$ $\quad p_2 = \begin{pmatrix} a & b & c \\ c & a & b \end{pmatrix}$

$c\ a\ b$

$p_1 \circ p_2 = \begin{pmatrix} a & b & c \\ b & c & a \end{pmatrix} \circ \begin{pmatrix} a & b & c \\ c & a & b \end{pmatrix} = \begin{pmatrix} a & b & c \\ a & b & c \end{pmatrix} = e$

$p \circ >$

(invertion)

(swap) swap

$a\ b \rightsquigarrow b\ a \rightsquigarrow a\ b$

$S \circ S = e$

$e = \begin{pmatrix} a & b \\ a & b \end{pmatrix}$ in $\begin{pmatrix} a & b \\ b & a \end{pmatrix}$

$a\ b \xrightarrow{\text{(identity)}} a\ b$

e

$f \circ g$ $\quad$ $g \circ f$

$(x+1)^2 \neq x^2+1$

Generally

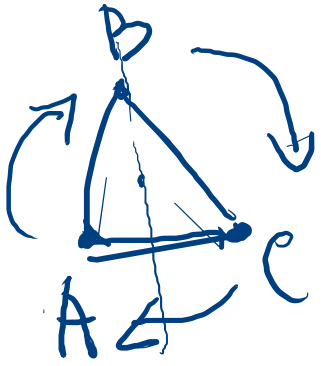
$$P_i \circ P_j \neq P_j \circ P_i$$

f = Square

g = plus 1

Symmetry

$\begin{pmatrix} ABC \\ CAB \end{pmatrix} \begin{pmatrix} ABC \\ CBA \end{pmatrix}$



regular

$f \circ g \neq g \circ f$