Epistemology of Topological Data Analysis

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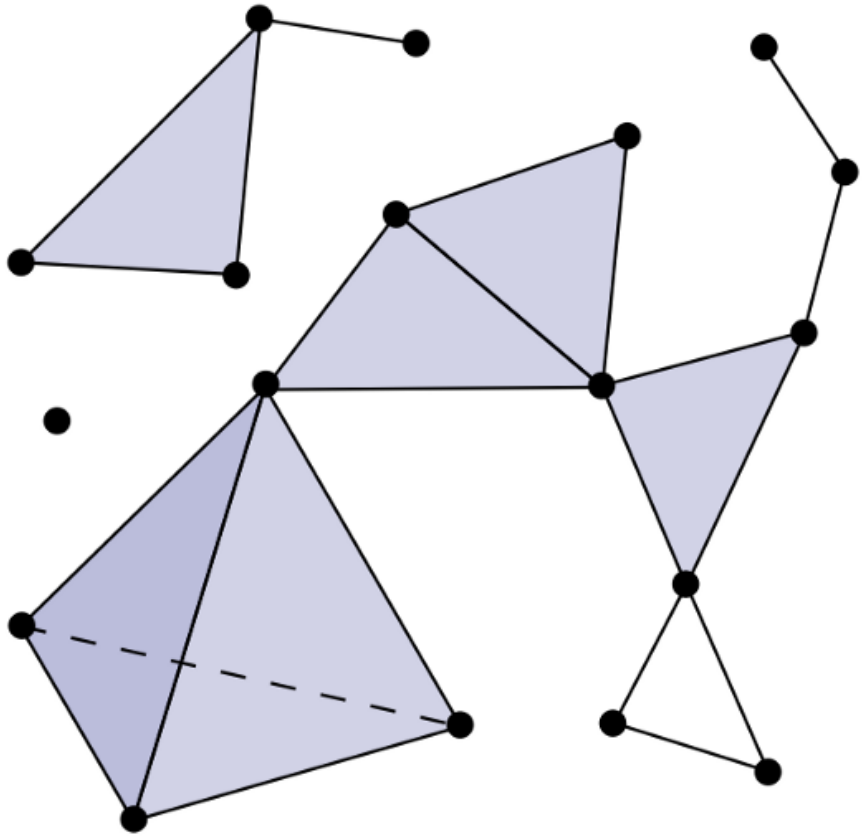
Homological Algebra

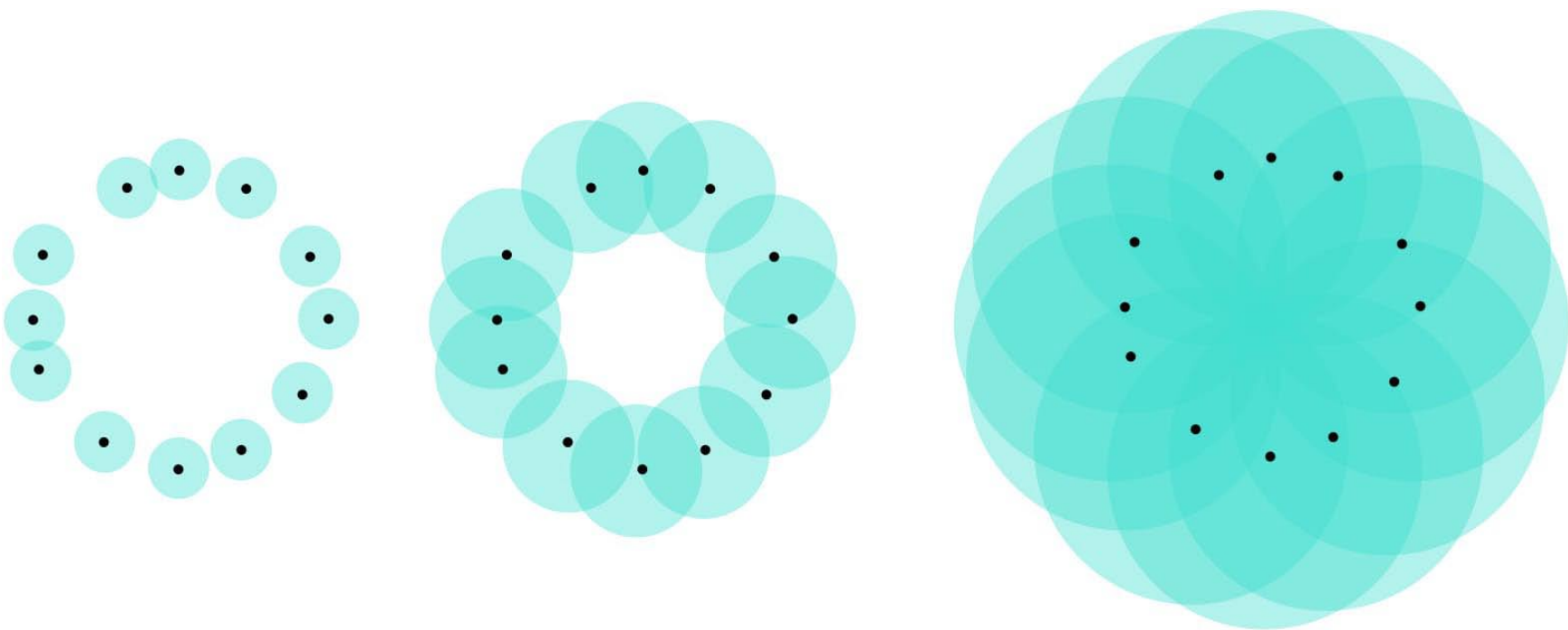
Simplicial complexes are generalized polyhedra:



Betti Numbers

- b_0 is the number of connected components;
- b_1 is the number of 1-dimensional "circular" holes;
- b_0 is the number of 2-dimensional holes aka cavities;
- ...
-
- b_0 is the number of k-dimensional holes.

Persistent homology (1990s, Morse theory)



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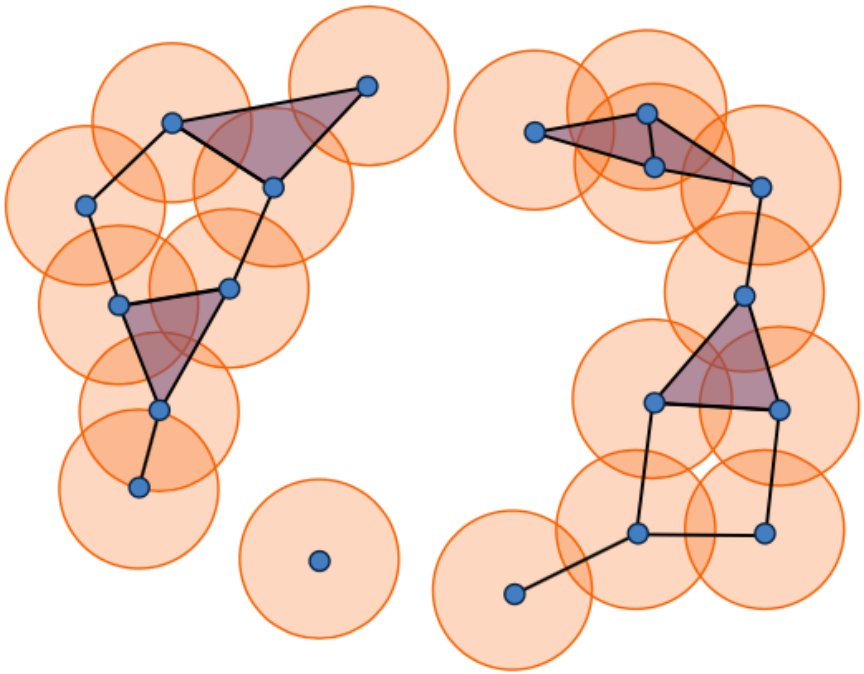
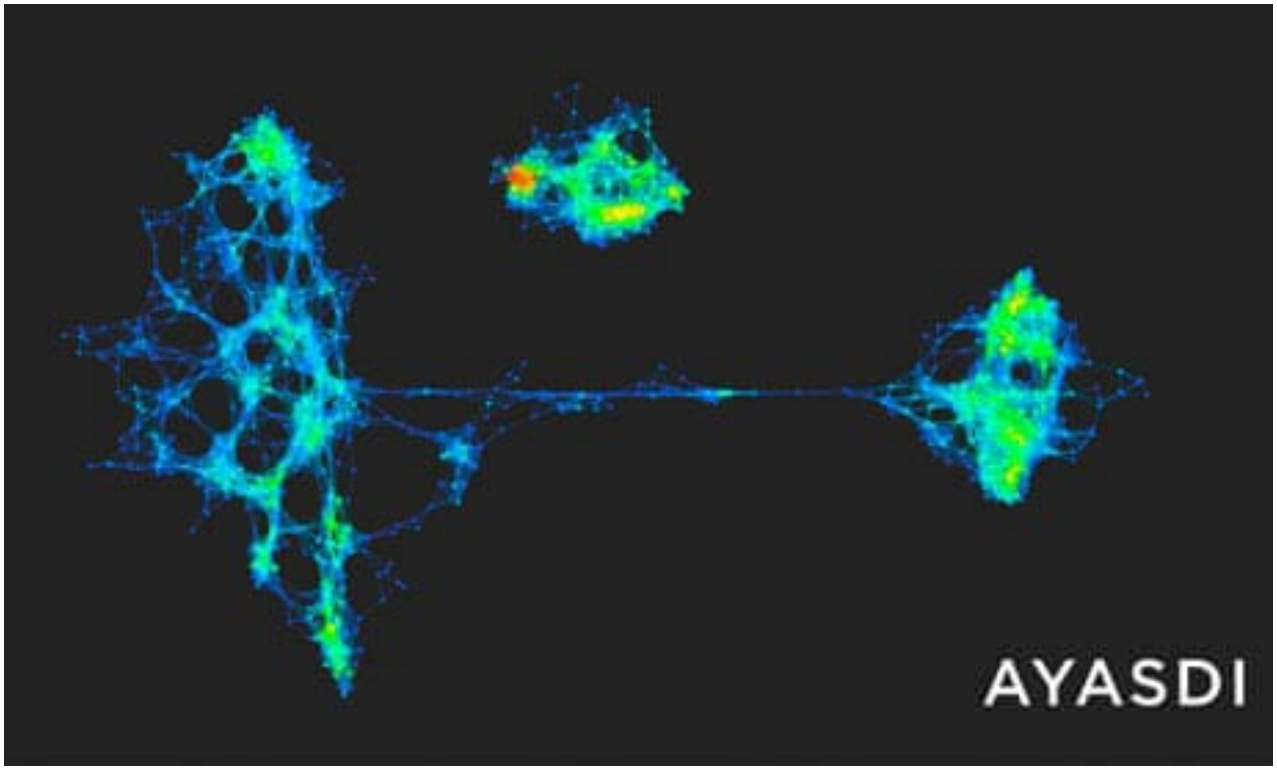


Topological profile of a dataset



What a topological profile says about the data?

Nobody, to the best of my knowledge, has an exact answer, however it is clear that this is quite a lot!

Voevodsky on Mathematics and Life

- Elementary Mathematics ($>$ 2000 y.) is integrated into the everyday life;
- "Higher" Mathematics (since 17 c.) is integrated into most sciences;
- "Modern" Mathematics (after 1850s) is integrated into some sciences;
- Synthetic Mathematics (after 1950s) is very poorly integrated (if at all).

End of Theory?

The End of Theory: The Data Deluge Makes the Scientific Met... http://www.wired.com/print/science/discoveries/magazine/16-0...

WIRED

<< Back to Article

WIRED MAGAZINE: 16.07

The End of Theory: The Data Deluge Makes the Scientific Method Obsolete

By Chris Anderson 06.23.08

Illustration: Marian Bantjes

"**All models are wrong**, but some are useful."

Chris Anderson 2008

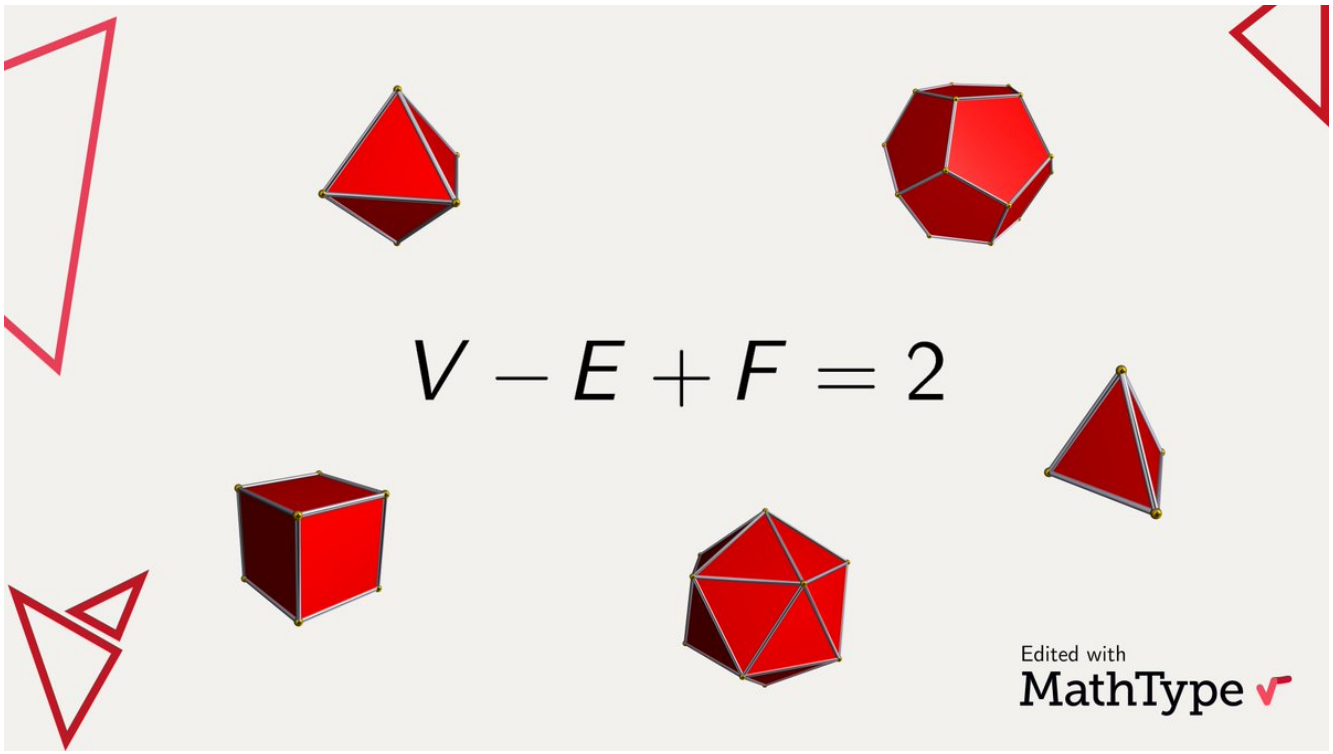
The new availablility of huge amounts of data, along with statistical tools to crunch these numbers, offers a whole new way of understanding the world. Correlation supersedes causation, and science can advance even without coherent models, unified theories, or really any mechanical explanation at all. There is no reason to cling to our old ways. It's time to ask: What can science learn from Google?

Yu.I Manin 2013

What can science learn from Google?: **Think!** Otherwise no Google will help you.

Euler 1750-1758

How it happened that nobody (with minimal reservations) discovered Euler Characteristic of polyhedra before Euler?



acies vocabo

A part of the answer is that the concept of *edge* (of a polyhedron) did not exist before Euler introduced it:

§. 6. Ad ambitum autem cuiusque ſolidi figuris planis inclusi pertinent 1^mo ipsae figurae planae eius ambitum constituentes, quae hedrae vocantur; 2^do binarum hedrarum secundum latera concursus, quibus termini lineares solidi oriuntur: hos terminos, quoniam apud scriptores Stereometriae nullum nomen proprium reperio, acies vocabo; 3^tio puncta, in quibus tres plureue hedrae concurrunt, quae puncta anguli solidi appellantur.

TDA and the "brute computational force"

Concept-based TDA combined with Machine Learning is more effective than the "brute" ML (whatever it may mean) alone!.

Robert May 2004

A paradigmatic account of the uses of mathematics in the natural sciences comes, in deliberately oversimplified fashion, from the classic sequence of Brahe, Kepler, Newton: observed facts, patterns that give coherence to the observations, fundamental laws that explain the patterns [. . .].

Consider the role played by applications of mathematics in sequencing the human and other genomes [. . .]. The sequence information, however, represents only the Tycho Brahe stage.

Current work on various genomes uses pattern-seeking programs to sort out coding sequences corresponding to individual genes [. . .]. Again, elegant and sometimes novel mathematics is involved in this Keplerian stage of the work in progress.

Robert May 2004

We are only just beginning, if that, the Newtonian stage of addressing the deeper evolutionary questions posed by these patterns.

What the conceptual optics is not

A successful "local" application of certain mathematical concepts in science. Ex. topological motivations (made into a part of the theory in 1984 by Penrose&Rindler) of Paul Dirac's theory of electron (spin).

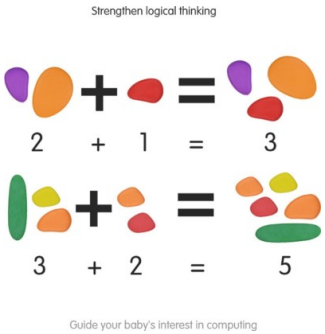
Remark: this is the context in which the problem of Wigner 1960 typically arises.

What the conceptual optics is

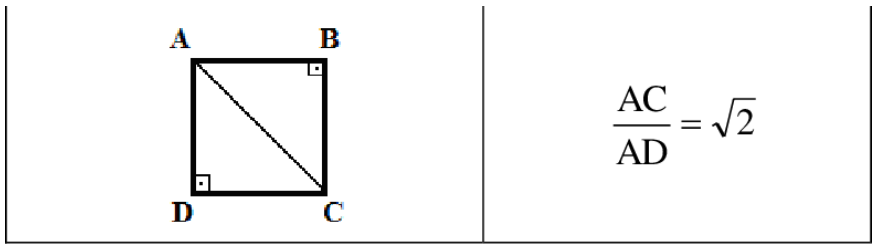
Its characteristic feature is **universality**.

Basic elements of human (mathematical) conceptual optics

(natural) number

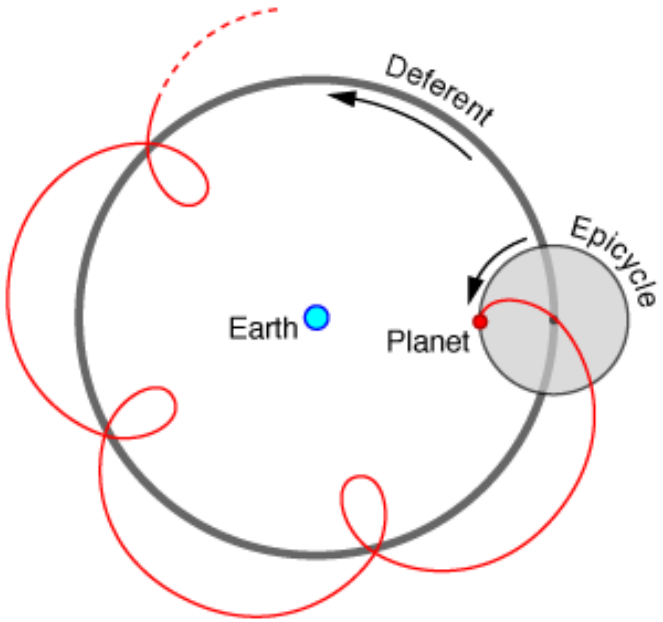


measure and magnitude



Euclidean optics: epicycles

Eiuclid, Elements, c. 300 B.C.
(ср. разложение в ряд Фурье)



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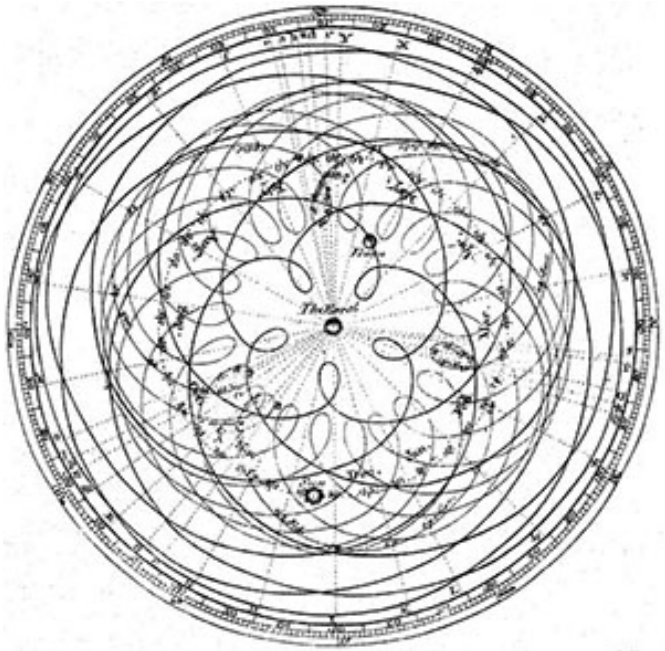
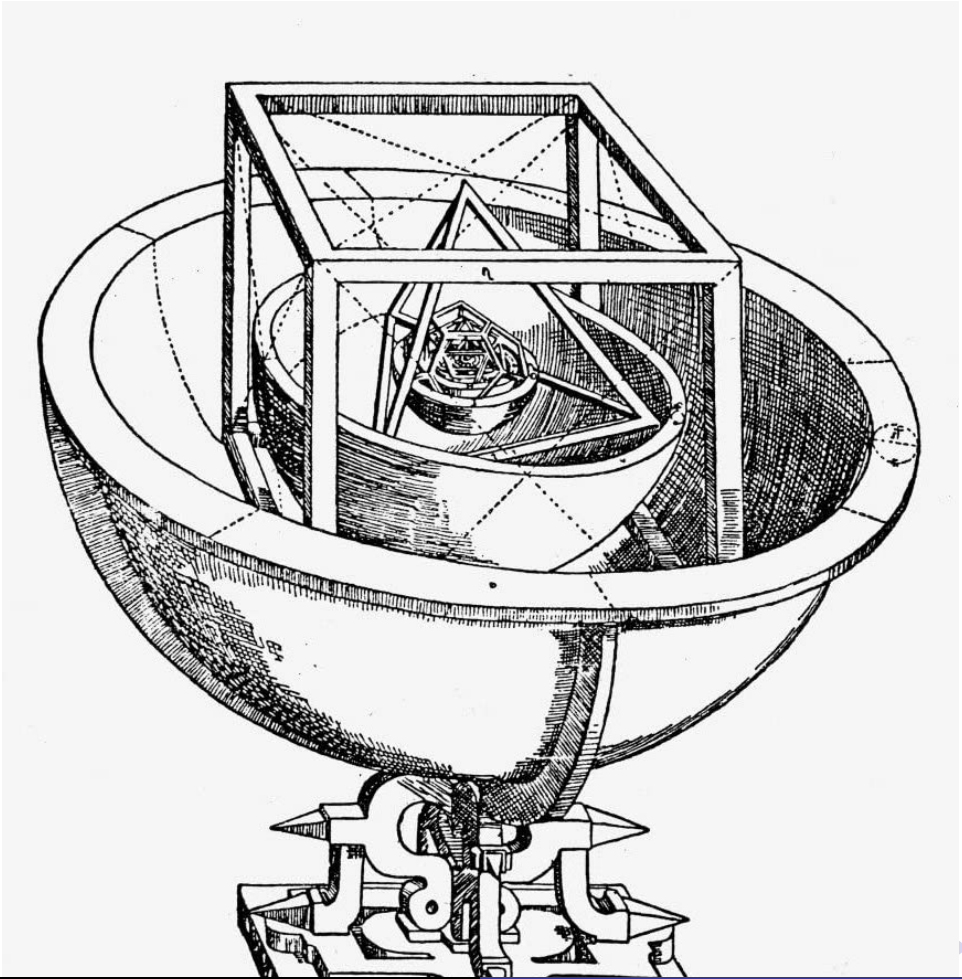


D. Park : In the midst of all this empiricism sat the ghost of Plato, legislating that the curves drawn must be circles and nothing else, and that the planets and the various connection points must move along them uniformly and in no other way.

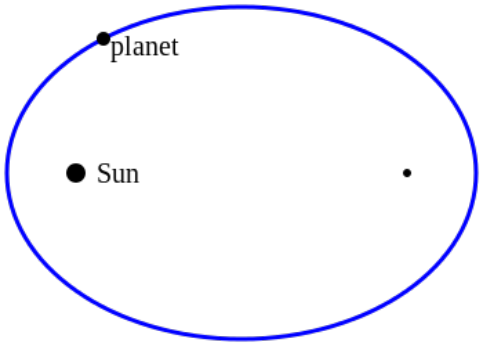
Kepler 1596

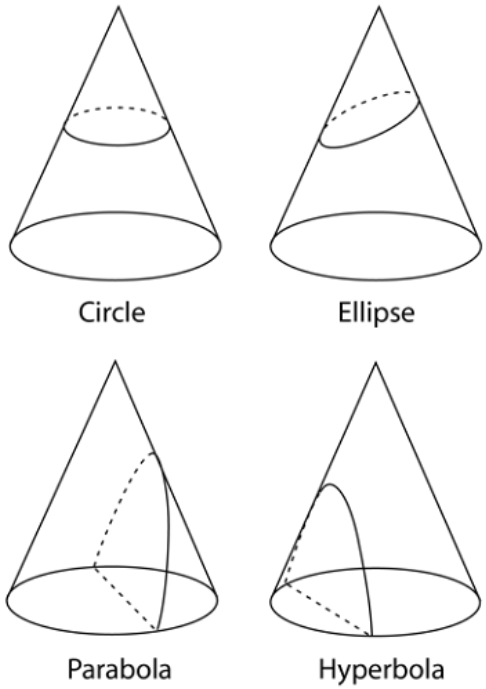
Mysterium Cosmographicum:



Kepler 1607

Astronomia nova Aitiologitis 1607:



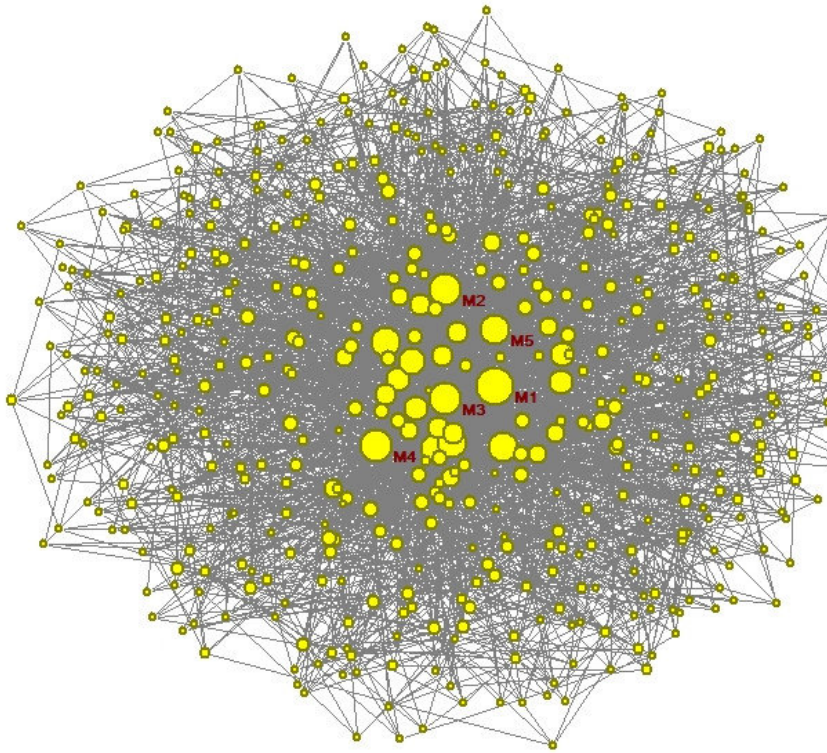


Newtonian optics 1671: fluentae et fluxiae

(published posthumously in 1736)

Those Quantities which I consider as gradually and indefinitely increasing I shall hereafter call Fluents or Flowing Quantities... The velocities by which every Fluent is increased by its generating Motion I may call Fluxions or simply Velocities or Celerities.

the old and the new image of the stock market



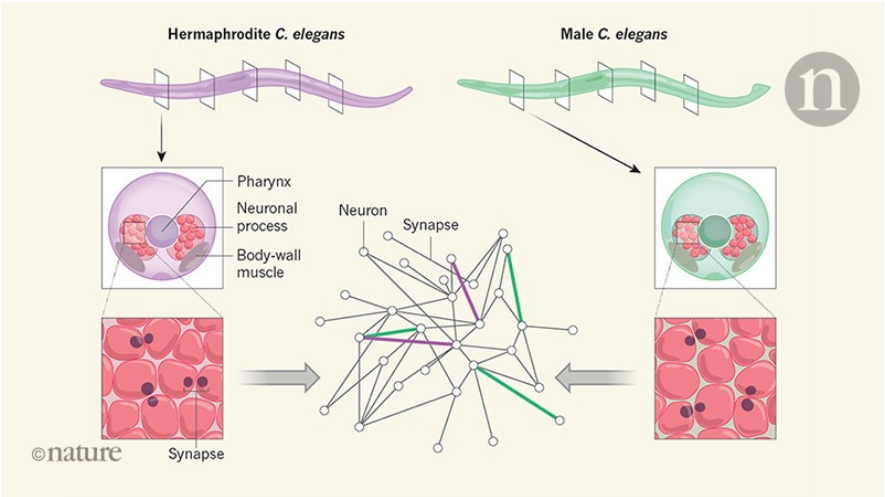
Kepler and TDA

TDA meets the universality criterion and qualifies as a conceptual *optics* in the above.

However, the lack of stable explanatory patterns allows to qualify TDA (after May 2004) as Keplerian but not yet as Newtonian mathematical tool/foundation for science.

Neuroiscience / Brain Research as a new paradigm for science?

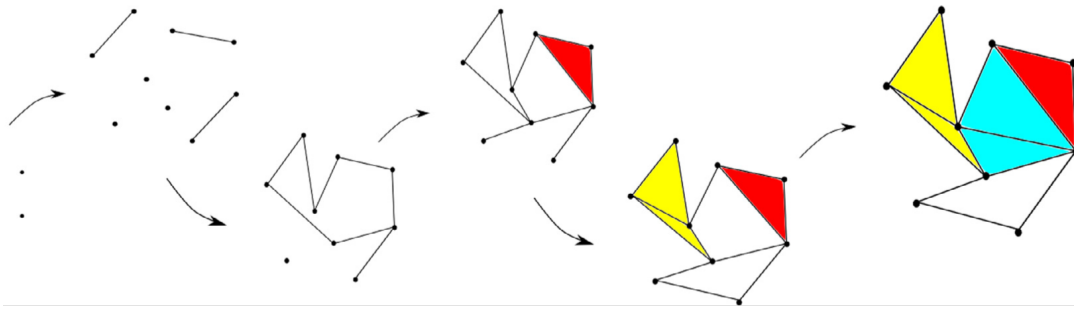
from the *connectom* (Nature 2019)



to . . .

Scaling of connective structures

simplicial complexes



Conclusions:

(1) TDA is a rear example of successful application of recently developed abstract mathematical concepts in a large variety of empirical studies;

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(2)TDA is a new candidate mathematical conceptual optics, which is based on the 300 years long continuing history of topological reasoning;

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(3) Neurobiology and Brain Science provide new patterns of "topological" explanation, which needs to be sharped and studied from an epistemological viewpoint; these patterns of scientific explanation may prove to be relevant in other data-driven research areas such as Climate Research, Economics, etc.;

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(4) Paraphrasing Kant : Berechnungen ohne Begriffe sind blind (und also langweilig ☹), Begriffe ohne Berechnungen sind steril.

СПАСИБО!