Scientific Progress and Rewriting of History

Andrei Rodin

Workshop Ontological and Epistemological Aspects of Textual Sources and Resources, MSH Lorraine

July 11, 2023

Plan:

1. Two Modes of Transmission of Textual Sources
2. Euclid and His Modern Rivals
3. The Pythagorean Theorem from Euclid to Einstein
4. Identity of mathematical and scientific contents through change and through historical time
5. Scientific Progress (instead of Conclusion)

1 Two Modes of Transmission of Textual Sources

2 Euclid and His Modern Rivals

3 The Pythagorean Theorem from Euclid to Einstein

4 Identity of mathematical and scientific contents through change and through historical time

5 Scientific Progress (instead of Conclusion)

Thomas Kuhn on Rewriting of History

[T]o an extent unprecedented in other fields, both the layman's and the practitioner's knowledge of science is based on textbooks and a few other types of literature derived from them. Textbooks, however, being pedagogic vehicles for the perpetuation of normal science, have to be *rewritten* in whole or in part whenever the language, problem-structure, or standards of normal science change. [...]

Thomas Kuhn on Rewriting of History

[T]o an extent unprecedented in other fields, both the layman's and the practitioner's knowledge of science is based on textbooks and a few other types of literature derived from them. Textbooks, however, being pedagogic vehicles for the perpetuation of normal science, have to be *rewritten* in whole or in part whenever the language, problem-structure, or standards of normal science change. [...]

Textbooks thus begin by truncating the scientist's sense of his discipline's history and then proceed to supply a substitute for what they have eliminated. [...] [T]he textbook-derived tradition in which scientists come to sense their participation is one that, in fact, never existed. [...]

Thomas Kuhn on the Rewriting of History

No wonder that textbooks and the historical tradition they imply have to be *rewritten* after each scientific revolution. And no wonder that, as they are *rewritten*, science once again comes to seem largely cumulative. [...]

Thomas Kuhn on the Rewriting of History

No wonder that textbooks and the historical tradition they imply have to be *rewritten* after each scientific revolution. And no wonder that, as they are *rewritten*, science once again comes to seem largely <u>cumulative</u>. [...]

The depreciation of historical fact is deeply, and probably functionally, ingrained in the ideology of the scientific profession. [...] The result is a persistent tendency to make the history of science look linear or <u>cumulative</u>.

Thomas Kuhn on Scientific Progress (in my own words)

A cumulative grow of scientific knowledge (that is, scientific progress) through revolutions is illusionary.

The notion of scientific progress (= cumulative grow of knowledge along some identifiable group of parameters) to historical fact only during short periods of "normal" science. Every scientific revolution introduces new epistemic standards and hence new parameters, and reconstructs the past developments producing an *illusion* of continuous scientific progress from pre-revolutionary times to today. (Cf. Soviet historiography; Orwellian Ministry of Truth)

Two modes of transmission of knowledge (and other contents) through time with texts (alphabetic, symbolic, diagrammatic):

Traditional (?) mode

Preservation of original *Urtexts* up to literal copying (and eventually translations into other natural languages), building a learning tradition and a commentary tradition (glosses, scholia etc.).

Traditional (?) mode

Preservation of original *Urtexts* up to literal copying (and eventually translations into other natural languages), building a learning tradition and a commentary tradition (glosses, scholia etc.).

Applies in world religions (ex. Bible), literature prose and poetry, philosophy (where the corpus of Urtexts depends on philosophical school, ex. Analytic Philosophy)

Scientific mode

Systematic rewriting of textual sources preserving the content (?) but its textual bearer.

Scientific mode

Systematic rewriting of textual sources preserving the content (?) but its textual bearer.

Applies in Science and Mathematics. (Allegedly) is compatible with / supports a progressive development.

1 Two Modes of Transmission of Textual Sources

2 Euclid and His Modern Rivals

3 The Pythagorean Theorem from Euclid to Einstein

4 Identity of mathematical and scientific contents through change and through historical time

5 Scientific Progress (instead of Conclusion)

Example: Euclid's *Elements*

Bible of Mathematics?

Example: Euclid's *Elements*

Bible of Mathematics? Very far from it!

John Murdoch (1927 - 2010) on the Transmission of Euclid's Elements

Any attempt to plot the course of Euclid's Elements from the third century B.C. through the subsequent history of mathematics and science is an extraordinary difficult task. No other work - scientific, philosophical, or literary - has, in making its way from antiquity to the present, fallen under an editor's pen with anything like an equal frequency. And with good reason: it served, for almost 2000 years, as the standard text of the core of basic mathematics. As such, the editorial attention it constantly received was to be expected as a matter of course.

Urtext

Euclidis Opera omnia. Ediderunt I. L. Heiberg et H. Menge. (Lipsiae, B. G. Teubneri, 1883-1916)
Theon of Alexandia (c. 335 - 405 A.D.) –> Simon Grynaeus (Basel 1533): *editio princeps*

Theon in Commentary on Almagest : "last sentence of El. 6 is added by myself".

Francois Peyrard 1808 discovers in Vatican Library a MS without this additional sentence; Heiberg makes his critical edition on this basis

M. Klamroth: some Arab translations are based on more reliable sources.

Transmission Overview

- Euclid's Akmé is between 347 B.C. (Plato's death) and 287 B.C. (Archimedes' birth);
- Greek phase: before c. 400 (Theon);
- Roman/Early Medieval Greco-Latin phase including Boetius (480-524 н.э.);
- Arab phase: c. 800-1200 A.D.; Murdoch: "The Arabic phase of the Elements' history may well prove to be not merely the most manifold but, even mathematically, the most creative of all";
- Arab-Latin phase: c. 1100-1400;
- Renaissance / Early Modern Greco-Latin phase: c. 1400-1700
- Modern Latin/Vernacular phase: c. 1550-
- Since Heiberg's edition (c. 1900): separation of Euclid's geometry (the geometrical books of Euclid's *Elements*) form the Euclidean geometry (à la Hilbert, Bourbaki, etc.) — albeit still too often not sufficiently precise.

Transmission in 15-16 c.: a competition between the two modes

- Campanus of Novara (Venice 1482) and Bartalameo Zamberti (Venice 1505): a deep modernisation;
- Grynaeus (Basel 1533): *editio princeps* of Greek (Theon) original: had a minimal influence;
- Commandino: Urbino 1575 Latin based on Grynaeus' Greek;
- Clavius: Rome 1574: "not, properly speaking, a translation [...]ṙbut a personal reduction" (486 Euclid's propositions plus 671 new propositions, deep modernisation).

first translations into vernacular languages

- Italian: Tartaglia (Venice) 1543;
- English: Recorde 1551 “For nother is there anie matter more straunge in the englishe tungue, then this whereof never booke was written before now, in that tungue”;
- German: Scheubel 1558;
- French: Forcadel (Paris) 1564.

Transmission or Innovation?: 17-18 c.

- Andreé Tacquet: Antwerp 1654: Elementa geometriae planae et solidae (became "Euclid" in the Second edition of 1725 !);
- Isaac Barrow: Cambridge 1655: Euclidis Elementorum libri XV breviter demonstrati (authenticity via modernisation);
- Girolamo Saccheri: Milan 1733: Euclides ab omni naevo vindicatus (Euclid Freed of Every Flaw).

since 17: "New" Elements

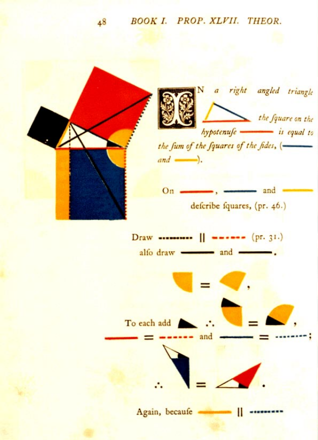
Antoine Arnauld 1667 and 1683: Nouveaux Eléments de géométrie

Transmission: 19 c.

Murdoch: “a veritable avalanche of Euclid primers, frequently radically divergent from any imaginable text of the Elements”

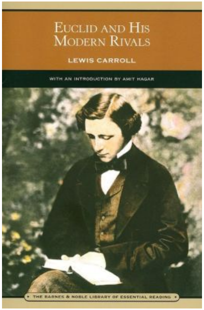
Transmission: 19 c.

Oliver Byrne, The First Six Books of the Elements of Euclid in which coloured diagrams and symbols are used instead of letters for the greater ease of learners, 1847



Transmission: 19 c.

Charles Lutwidge Dodgson (aka Lewis Carrol), Euclid and his Modern Rivals, London 1879



20th c.: some translations from Heiberg's text

- English: Thomas L. Heath, The thirteen books of Euclid's Elements (Cambridge: Cambridge University Press, 1908);
- Russian: Д. Д. Мордухай-Болтовской, Начала Евклида, т. 1-3, М.-Л.: ГИТТЛ 1948-1950;
- Bernard Vitrac, Euclide d'Alexandrie, Les Éléments, traduits du texte de Heiberg, P.U.F. 1994.

"New Elements" of the 20th century

- David Hilbert, Gundlagen der Geometrie 1899, 1903, ..
- Nicolas Bourbaki, Eléments de mathématique 1939 -

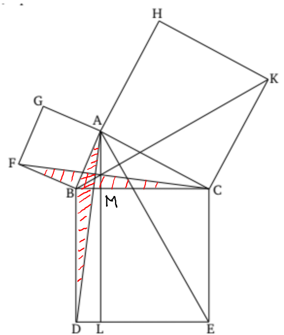
1. Two Modes of Transmission of Textual Sources
2. Euclid and His Modern Rivals
3. The Pythagorean Theorem from Euclid to Einstein
4. Identity of mathematical and scientific contents through change and through historical time
5. Scientific Progress (instead of Conclusion)

Elements 1.47

Proposition (EP): $\square BCED = \square BFGA + \square AHKC$
Proof: $\triangle FBC = \triangle BAD = \frac{1}{2}\square BFGA = \frac{1}{2}\sqsubset\!\sqsupset BDLM$;

hence $\square BFGA = \sqsubset\!\sqsupset BDLM$; similarly $\square AHKC = \sqsubset\!\sqsupset MCEL$.

$\sqsubset\!\sqsupset BDLM + \sqsubset\!\sqsupset MCEL = \square BCED = \square BFGA + \square AHKC \blacksquare$



Equality, Sum and Difference of Figures, according to Euclid

Κοιναὶ ἔννοιαι.

αʹ. Τὰ τῷ αὐτῷ ἴσα καὶ ἀλλήλοις ἐστὶν ἴσα.

βʹ. Καὶ ἐὰν ἴσοις ἴσα προστεθῇ, τὰ ὅλα ἐστὶν ἴσα.

γʹ. Καὶ ἐὰν ἀπὸ ἴσων ἴσα ἀφαιρεθῇ, τὰ καταλειπόμενά ἐστιν ἴσα.

δʹ. Καὶ τὰ ἐφαρμόζοντα ἐπ᾽ ἀλλήλα ἴσα ἀλλήλοις ἐστίν.

εʹ. Καὶ τὸ ὅλον τοῦ μέρους μεῖζόν [ἐστιν].

Common Notions

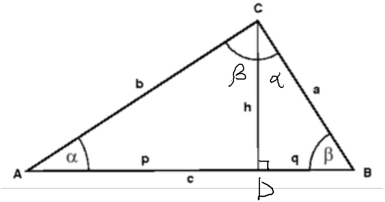
1. Things equal to the same thing are also equal to one another.
2. And if equal things are added to equal things then the wholes are equal.
3. And if equal things are subtracted from equal things then the remainders are equal.†
4. And things coinciding with one another are equal to one another.
5. And the whole [is] greater than the part.

my school geometry textbook by Kolmogorov et al.

Proposition (CP): $a^2 + b^2 = c^2$
Proof: $ACD = \beta; DCB = \alpha$ hence $\triangle ABC \sim \triangle ACD \sim \triangle DCB$. Let $AD = p$, $DB = a$, then $p + q = c$.

We have $\frac{a}{p} = \frac{p+q}{a}$, hence $a^2 = p^2 + pq$. Similarly $b^2 = q^2 + pq$.

$a^2 + b^2 = p^2 + 2pq + q^2 = [p + q]^2 = c^2$ ■



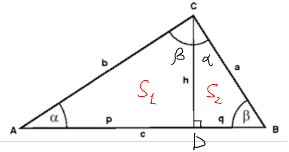
Einstein's Boyhood proof (*Saturday Review of Literature* November 1949)

$\triangle ABC \sim \triangle ACD \sim \triangle DCB$.
Let $S = Air[\triangle ABC], S_1 = Air[\triangle ACD], S_2 = Air[\triangle CBD]$,
so $S_1 + S_2 = S$

We have

$$\frac{S}{c^2} = \frac{S_1}{a^2} = \frac{S_2}{b^2}$$

Hence $S_1 b^2 = S_2 a^2$, $[S_1 + S_2]a^2 = S_1 c^2$. Hence $S_1 a^2 + S_1 b^2 = S_1 c^2$ and finally $a^2 + b^2 = c^2$ ■



Remarks:

Both Kolmogorov's and Einstein's proofs of the Pythagorean theorem are style of the 18th century mathematics (albeit Legendre follows Euclid more closely). More modern renderings of this theorem can be found in Bourbaki-style geometry textbooks.

Remarks:

Both Kolmogorov's and Einstein's proofs of the Pythagorean theorem are style of the 18th century mathematics (albeit Legendre follows Euclid more closely). More modern renderings of this theorem can be found in Bourbaki-style geometry textbooks.

The two versions of the Pythagorean theorem (EP and CP) are separated by the "symbolic revolution" that occurred in in mathematics in the 17th c. (Michel Serfati) and that brought into the Euclid-style geometry symbolic algebraic methods (Descartes 1937 et al.) and made CP possible.

Remarks:

Both Kolmogorov's and Einstein's proofs of the Pythagorean theorem are style of the 18th century mathematics (albeit Legendre follows Euclid more closely). More modern renderings of this theorem can be found in Bourbaki-style geometry textbooks.

The two versions of the Pythagorean theorem (EP and CP) are separated by the "symbolic revolution" that occurred in in mathematics in the 17th c. (Michel Serfati) and that brought into the Euclid-style geometry symbolic algebraic methods (Descartes 1937 et al.) and made CP possible.

Early elements of these methods are found already in the translation of Euclid's *Elements* by François Hérigone (1634) who applies to Euclid/Clavius' theory the novel symbolic approach earlier developed by François Viète's in his *In Artem Analyticam Isagoge* (1591).

1 Two Modes of Transmission of Textual Sources

2 Euclid and His Modern Rivals

3 The Pythagorean Theorem from Euclid to Einstein

4 Identity of mathematical and scientific contents through change and through historical time

5 Scientific Progress (instead of Conclusion)

Big Question

Is it still the same theorem?

Attempted answer:

YES because Euclid's geometry in general and Prop. 1.47 in particular *translates* into the algebraic symbolic language.

Attempted answer:

YES because Euclid's geometry in general and Prop. 1.47 in particular *translates* into the algebraic symbolic language.

This allows one to think of the Pythagorian theorem (not as a Platonic timeless truth about a certain timeless Platonic object but) as a developing theoretical entity that preserves its identity through the continuous change in the historical time without preserving the identity of its textual presentation (including the identity up to paraphrasing and other forms of linguistic equivalence).

Attempted answer:

YES because Euclid's geometry in general and Prop. 1.47 in particular *translates* into the algebraic symbolic language.

This allows one to think of the Pythagorian theorem (not as a Platonic timeless truth about a certain timeless Platonic object but) as a developing theoretical entity that preserves its identity through the continuous change in the historical time without preserving the identity of its textual presentation (including the identity up to paraphrasing and other forms of linguistic equivalence).

This global dynamics results from a more subtle local dynamics that comprises a variety of simultaneous revisions occurring in scholarly communications. This local dynamics has been so far very little studied.

BUT

Kuhn is quite right that presently there is no commonly accepted *criteria of faithfulness* or standard of translation of older contents into newer theoretical settings, which could justify the identity claims. Scientists and mathematicians too often make historical identity claims dating back the origins of their favourite theories to some noble past achievements — without supporting such claims with any sound historical and theoretical justification (just like populist politicians do).

But Kuhn is wrong when he derives from such critical observations nothing but sceptical conclusions.

BUT

Kuhn is right that the identity of mathematical and scientific contents through time (and hence the notion of progress) is a social and theoretical construction.

But for some reason Kuhn doesn't take seriously the possibility that such a construction can correspond to the historical fact, if it is made carefully and formulated properly. Such an identity-building is not a straightforward matter; it requires a lot of theoretical and a historical expertise. The task is impossible unless the old textual sources are reasonably well preserved, and an art of their faithful interpretation effectively transfers through generations.

Category theory

A modern mathematical instrument that may help to manage translations between scientific and mathematical contents is Category Theory (CT). Cf. Ulyses Moulines' attempt to save the notion of progress from the Kuhnean critique using Bourbaki-style model-theoretic approach.

Category theory

A modern mathematical instrument that may help to manage translations between scientific and mathematical contents is Category Theory (CT). Cf. Ulyses Moulines' attempt to save the notion of progress from the Kuhnean critique using Bourbaki-style model-theoretic approach.

I do believe that CT sheds a new light on the metatheoretical issue of Theory Change in mathematics and elsewhere, but I also realise that like any other formal tool it requires to prepare or "formalise" the concerned contents (say, EP and CP) in advance, and this operation is not, generally, quite innocent. Formal tools can help but *not replace* a properly historical expertise.

1 Two Modes of Transmission of Textual Sources

2 Euclid and His Modern Rivals

3 The Pythagorean Theorem from Euclid to Einstein

4 Identity of mathematical and scientific contents through change and through historical time

5 Scientific Progress (instead of Conclusion)

The non-invertibility of translations through the historical time

However the notion of translation EP→CP is theoretically refined, it is clear that it is not invertible: one can, by and large, express Euclid's geometry in the language of algebra (in fact, in many different ways and with different outcomes) but not all relevant algebraic contents can be translated back into the language of Euclid's geometry of ruler and compass!

New Knowledge via Translation: Impossible Problems

Pierre Wanzel 1837 (example of problem that could be formulated but not resolved in the Euclidean setting)

Supposons qu'un problème de Géométrie puisse être résolu par des intersections de lignes droites et de circonférences de cercle : si l'on joint les points ainsi obtenus avec les centres des cercles et avec les points qui déterminent les droites on formera un enchaînement de triangles rectilignes dont les éléments pourront être calculés par les formules de la Trigonométrie ; d'ailleurs ces formules sont des équations algébriques qui ne renferment les côtés et les lignes trigonométriques des angles qu'au premier et au second degré ; ainsi l'inconnue principale du problème s'obtiendra par la résolution d'une série d'équations du second degré dont les coefficients seront fonctions rationnelles des données de la question et des racines des équations précédentes. D'après cela, pour reconnaître si la construction d'un problème de Géométrie peut s'effectuer avec la règle et le compas, il faut chercher s'il est possible de faire dépendre les racines de l'équation à laquelle il conduit de celles d'un système d'équations du second degré composées comme on vient de l'indiquer. Nous traiterons seulement ici le cas où l'équation du problème est algébrique.

New Knowledge via Translation: Impossible Problems (contd)

Il résulte immédiatement du théorème précédent que tout problème qui conduit à une équation irréductible dont le degré n'est pas une puissance de 2, ne peut être résolu avec la ligne droite et le cercle. Ainsi *la duplication du cube*, qui dépend de l'équation $x^3 - 2a^3 = 0$ toujours irréductible, ne peut être obtenue par la Géométrie élémentaire. Le problème *des deux moyennes proportionnelles*, qui conduit à l'équation $x^3 - a^2 b = 0$ est dans le même cas toutes les fois que le rapport de b à a n'est pas un cube. La *trisection de l'angle* dépend de l'équation $x^3 - \frac{3}{4} x + \frac{1}{4} a = 0$; cette équation est irréductible si elle n'a pas de racine qui soit une fonction rationnelle de a et c'est ce qui arrive tant que a reste algébrique ; ainsi le problème ne peut être résolu en général avec la règle et le compas. Il nous semble qu'il n'avait pas encore été démontré rigoureusement que ces problèmes, si célèbres chez les anciens, ne fussent pas susceptibles d'une solution par les constructions géométriques auxquelles ils s'attachaient particulièrement.

Claim (seems rather obvious but so far remains conjectural): Wanzel's argument canNOT be reasonably translated back to Euclid's geometrical language.

Progress as a partial order structure

Assuming that there is at most one translation $t : T_1 \to T_2$ between a theoretical entities T_1 and its successor T_2 that represent different forms or "temporal stages" of the same theoretical entity T , and identifying isomorphic objects as usual, one gets a partial order structure that suffices for supporting a weak notion of progress as cumulative growth, see the above example. Recall that the partial order structure is the core form of the part/whole relation.

Progress as a partial order structure

Assuming that there is at most one translation $t : T_1 \to T_2$ between a theoretical entities T_1 and its successor T_2 that represent different forms or "temporal stages" of the same theoretical entity T , and identifying isomorphic objects as usual, one gets a partial order structure that suffices for supporting a weak notion of progress as cumulative growth, see the above example. Recall that the partial order structure is the core form of the part/whole relation.

A stronger (and, to my sense, exceedingly strong) notion of progress requires that the partial order is such that every older theory has at least one today's successor, so "no older knowledge is lost".

Progress as a partial order structure

Assuming that there is at most one translation $t : T_1 \to T_2$ between a theoretical entities T_1 and its successor T_2 that represent different forms or "temporal stages" of the same theoretical entity T , and identifying isomorphic objects as usual, one gets a partial order structure that suffices for supporting a weak notion of progress as cumulative growth, see the above example. Recall that the partial order structure is the core form of the part/whole relation.

A stronger (and, to my sense, exceedingly strong) notion of progress requires that the partial order is such that every older theory has at least one today's successor, so "no older knowledge is lost".

CT allows for describing by far more refined structures of theoretical dynamics, progressive or not.

Rewriting and Science/Mathematics Education

Unless older contents are systematically reformulated, represented and materially rewritten in a more compact form, there is no way that new generations of scientists and mathematicians could learn the ever growing body of existing knowledge. For human learning capacities do not change as rapidly (if at all) as our mathematics, science and technology.

Rewriting and Science/Mathematics Education

Unless older contents are systematically reformulated, represented and materially rewritten in a more compact form, there is no way that new generations of scientists and mathematicians could learn the ever growing body of existing knowledge. For human learning capacities do not change as rapidly (if at all) as our mathematics, science and technology.

The revision of older contents and rewriting of their representations not only supports progress as just explained but also helps to comply with the anthropological limit of individual learning capacities.

Conclusion 1

Scientific Progress is a highly complex and also highly vulnerable social institution. A novelty or "innovation" does not guarantee progress by itself because the "established" knowledge is not preserved automatically in the "Third World" of Karl Popper or elsewhere.

Progress implies a cumulative development (in accordance with Kuhn), which, in its turn, requires a transmission of the older contents to new generations through the continuous revision, continuous rewriting and continuous compactification of older textual sources.

Conclusion 2

Science and mathematics do not transmit their contents by preserving the identities of these contents via a recurrent reference to a single limited set of Urtexts fixed once and for all.

But the progressive development of science and mathematics require to preserve not less but more texts. In order to construct an identity of scientific or mathematical theory that performs a progress, one needs to take into consideration a historically continuous chain (or network) of works that may justify such identity claims via reference to firm historical facts and their (re)interpretations, and not by establishing merely theoretical links between current theories and some arbitrarily chosen historical materials.

Conclusion 3

In order to promote the progress of science and mathematics in the future, we need to think today not only how to further preserve available historical sources but also how to improve (if not to guarantee) the long-term preservation of traces of our current scientific and mathematical communications.

THANKS!