Symposium: Proofs and styles of reasoning across history and cultures

Topic: C.1. Philosophy of the Formal Sciences (including Logic, Mathematics, Statistics)

When: Friday, July 28, 2023 from 9h to 10h30 and from 11h to 13h

Where: School of Economics of the University of Buenos Aires Av. Córdoba 2122, City of Buenos Aires, Argentina, Room 3, as a part of the CLMPST 2023.

Programme:

9h Palomäki, Jari (Tampere University), On Uuno Saarnio´s Attempted Proof of the Continuum Hypothesis

9h30 Rodin, Andrei (University of Lorraine), Proofs and Solutions, according to Kolmogorov

10h30Stern, Julio (USP - Universidade de Sao Paulo), Symmetry and Proof in Physics and Statistics: The Meaning of Nöther and deFinetti Theorems

10h30 - 11h Coffee

11h Vandoulakis, Ioannis (The Hellenic Open University), Analysing proving discourse: a dialogical perspective

11h30 Centrone, Stefania (Fern-Uni Hagen), Conceptions of Proof from Aristotle to Gentzen's Calculi

12h00 Mainzer, Klaus (Technical University of Munich), Proof and Computation in Logic, Mathematics, and Artificial Intelligence

12h30 Friend, Michele (George Washington University, Université de Lille Nord-Europe), The Transcendental Truth-Value in Some Buddhist Logic

Abstracts (in the order of talks):

(1) Title: On Uuno Saarnio´s Attempted Proof of the Continuum Hypothesis

Authors: Palomäki, Jari (Tampere University)

Abstract: In 1953, the Finnish mathematician and philosopher Uuno Saarnio (1896-1977) got acquainted with German mathematician Heinrich Behmann (1890-1970) at the XIth International Congress of Philosophy in Brussells, Belgium. Before that, Saarnio had already made research on transfinite ordinal numbers as well as twice attempted to show by means of them the correctness of the continuum hypothesis, but without success, [1][2]. However, now he had found a collaborator, an adviser, a referee, and a friend for his research, which culminated in a book Das System und die Darstellung der transfiniten Ordnungszahlen mit Hilfe der höheren Rechenoperationen. Mit Einführung von Prof. Dr. Heinrich Behmann by Saarnio in 1958, [3]. Based on that research, the main focus of it being higher-order counting laws for transfinite ordinal numbers, Saarnio published several articles in the 1960s on transfinite ordinal numbers especially in Mathematische Annalen, including his third and last attempt to prove the correctness of the continuum hypothesis in 1968: “Eine konstruktive Darstellung für die Richtigkeit der Kontinuumhypothese,” [4]. Thus, it took almost ten years of intensive study, criticism, meetings and correspondence between Behmann and Saarnio before Behmann, at last, was convinced of its correctness. Since when Behmann first heard Saarnio’s new attempted proof of the continuum hypothesis, he was very skeptical of it, mostly because of the independence proofs by Gödel and Cohen. When analysing Behmann’s struggle of Saarnio’s proof, I shall follow Joseph Coguen’s (1941-2006) idea of proof—events, e.g. [5], which presuppose at least two types of agents: prover, i.e. Saarnio, and interpreter, i.e. Behmann.

References:

[1] Saarnio, U.: “Ylinumeroituva hyvinjärjestys.” Ajatus, 236-261, (1944)
[2] Saarnio, U.: Die Wohlordung einer nichtabzählbaren Menge und die Lösung des Kontinuumsproblems. Helsinki: Gesellschaft für Logik und Ihre Anwendungen. (1953)
[3] Saarnio, U.: Das System und die Darstellung der

transfiniten Ordnungszahlen mit Hilfe der höheren Rechenoperationen. Mit Einführung von Prof. Dr. Heinrich Behmann. Helsinki: Gesellschaft für Logik und Ihre Anwendungen. (1958)
[4] Saarnio, U.: "Eine konstruktive Darstellung für die Richtigkeit der Kontinuumhypothese." Mathematische Annalen 178. 335-353. (1968)
[5] Stefaneas, P., Vandoulakis, I.: "Proofs as Spatio-Temporal Processes." Philosophia Scientiae. 111-125. (2014) https://journals.openedition.org/philosophiascientiae/1010

(2) Title: Proofs and Solutions, according to Kolmogorov
Authors: Rodin, Andrei (University of Lorraine)

Abstract: The popular BHK-semantics (after the names of Brouwer, Heyting and Kolmogorov) aka proof-interpretation of intuitionistic logic was first introduced under this name by Troelstra and van Dalen in the 1980s [1]. It deliberately combined a number of more specific interpretations of the same formal calculus earlier proposed by the aforementioned and some other people (including G. Kreisel). This synthesis was realized by Troelstra and van Dalen on the assumption that the differences between these more specific interpretations, in their analysis, were superficial and in any event not logical. Andrei N. Kolmogorov, however, held a different opinion, and considered his interpretation of the intuitionistic propositional logic as the "calculus of problems" to be essentially different from Arend Heyting's original interpretation of this calculus as a variety of propositional logic [2]. In Kolmogorov's view this difference had important epistemological implications, which Kolmogorov stressed at many occasions, in particular, in his Preface to Russian translation of Heyting's 1934 monograph, which appeared in 1936. [3,4]

A key point where the two mathematicians disagreed was whether every problem reduces to a proposition and, by consequence, whether every solution reduces to a proof. While Heyting understood the concept of proposition (Germ. Aussage) after Brouwer so broadly that he could call by this name every mathematical problem, Kolmogorov insisted that problems and propositions were two related but nevertheless sharply distinct notions.

In this talk I explain the differences between Kolmogorov's and Heyting's conceptions of the intuitionistic logic and show that Kolmogorov's conception can be combined with Heyting's only at the price of a very significant simplification or even a trivialisation of the former. Further, I defend Kolmogorov's view on problems and propositions (resp. solutions and proofs) using some historical and some recent mathematical examples including insights from Homotopy Type theory.

Bibliography:
1. M. van Atten, "The Development of Intuitionistic Logic", in: E.N. Zalta (ed.), The Stanford Encyclopedia of Philosophy, https://plato.stanford.edu/archives/sum2022/entries/intuitionistic-logic-development/ , last viewed December 2022
2. A.N. Kolmogorov, "Zur Deutung der Intuitionistischen Logik", Mathematische Zeitschrift, 35 (1932), p. 58–65
3. A. Heyting, Mathematische Grundlagenforschung, Intuitionismus, Beweistheorie, Berlin: Springer 1934
4. Russian translation of [3] by A.P. Yushkevich with Preface by A.N. Kolmogorov, Moscow-Leningrad 1936

(3) <u>Title</u>: Symmetry and Proof in Physics and Statistics: The Meaning of Nöther and deFinetti Theorems
Authors: Stern, Julio (USP - Universidade de Sao Paulo)

Abstract: The Han glyph or character xiang (Mandarin) or katachi (Japanese) is used as a translation to the western word symmetry (i.e., equal-measure). The Han glyph depicts the same basic idea, in a slightly more elaborate form: It displays two tree trunks of the same size, followed by three strands of hair, that convey the idea of an external manifestation of an internal power or property. This idea of external manifestation of an internal power or property raises the question: Internal to what or to whom? Internal to the observed object, or internal to the observing subject? Physics and Statistics offer answers to this question in the form of Nöther and deFinetti theorems, relating symmetry conditions of a system to its invariant quantities and parameters. Yet, conflicting objective/ subjective interpretations for the source of the symmetry condition remain possible. In this paper we present very simple but detailed versions of these theorems, and discuss their conflicting interpretations.

(4) Title: Analysing proving discourse: a dialogical perspective

Authors: Vandoulakis, Ioannis (The Hellenic Open University)

Abstract: Alongside the traditional concept of proof, as establishing facts corresponding to truth, we adopt the meta-methodological concept of proof-event (or proof-instance or inference-instance) conceived to cover all types of proving. Proof events are initiated by the statement of a fixed problem (specified by certain conditions) and form sequences evolving in space and time, representing the proof's history. They are viewed as the interaction of at least two types of agents:

a) A prover, which can be a human or a machine or a combination of them (in the case of hybrid proving), and

b) an interpreter, which generally can be a human (or group of humans) or a machine (or group of machines) or a combination of them. [1].

Thus, proof events have a dialogical nature and generate proof narratives exposed in different styles that characterise individual provers, the schools they belong or the culture of which they are bearers.

The structure of proof narratives is organised in a complex hierarchical order. At the first level, expressions such as "definition" are used to introduce the contents (intentions) of a prover's mathematical thinking that direct the reader's (interpreter's) mind toward particular objects that possess "ontological status". At the second level, "assertions" represent states of affairs which possess "truth status". At the meta-linguistic level, expressions that do not refer to objects but linguistic entities used within the discourse are used. The combination of propositions into a proof step is made by using logical connectives. Furthermore, proof steps are combined to build proof represented in various styles that perform certain communicational functions.

Communication takes place between a prover and an (at least, potential) interpreter, who both participate in a (sequence of) proof-event(s), although they may be remote in space and time. By communicating a proof narrative, a prover addresses a (potential) "reader" (interpreter), expecting that he will read the information encoded in his proving outcome, understand (decode) it, and become persuaded that it is valid proof. The communicational and stylistic functions of (contemporary or

past) proof narratives can be examined using the six functions associated with the Jacobson communication model modified for proof events.

An interpreter's understanding of a prover's outcome is an active, dialogic process; an interpreter enters a "dialogue" with the prover, in which the interpreter may alter the initial proof by refining concepts, adding new concepts (definitions) or revealing and formalising implicit assumptions, filling possible gaps in the proof by proving auxiliary lemmas, theorems, etc. Thus, in some sense, the interpreter's activity is a reconstruction of meaning or conscious reproduction of the information content conveyed by the prover's outcome. In this context, we will reconsider the relevance of the hermeneutic legacy (Gadamer) and Russian formalism (Bakhtin's concept of dialogic imagination (chronotope)) for the discourse analysis of proving narratives.

References

[1] Stefaneas, P., Vandoulakis, I.M. 2014. "Proofs as Spatio-temporal processes", Philosophia Scientiae 18(3), 111-125.

(5) Title: Conceptions of Proof from Aristotle to Gentzen's Calculi

Authors: Centrone, Stefania (Fern-Uni Hagen)

Abstract: The talk aims to show how some key ideas at the basis of the normalization results in proof theory have their deep grounds in a number of fundamental questions that are posed always anew within the philosophical reflection on mathematics. Two different paradigms of proofs, synthetic and analytic, are contrasted and their origin is traced back to Aristotle as well as to Bernard Bolzano's idea of a better grounded presentation of mathematics at the beginnings of the 19th century.

(6) Title: Proof and Computation in Logic, Mathematics, and Artificial Intelligence

Authors: Mainzer, Klaus (Technical University of Munich)

Abstract: Proof assistants seem to open new avenues of research combining foundations of logic and mathematics with highly topical challenges of IT- and AI-technology. At first, we consider the development from the Curry-Howard correspondence of proofs and computer programs to the proof assistant Coq. Coq

is a platform for the verification of mathematical proofs as well as for the verification of computer programs in the calculus of inductive constructions (CiC). Finally, homotopy type theory (HoTT) allows mathematical proofs to be translated into a computer programming language of proof assistants even for advanced mathematical categories. The question arises whether, besides the verification of advanced mathematical proofs, advanced computer programs of (statistical) machine learning in AI can also be verified by proof assistants. The talk concludes with philosophical perspectives, practical challenges, and societal impact from verification to certification and responsibility of AI-programs.

References: K. Mainzer, P. Schuster, H. Schwichtenberg (eds.), Proof and Computation. Digitization in Mathematics, Computer Science, and Philosophy, World Scientific: Singapore 2018; K. Mainzer, P. Schuster, H. Schwichtenberg (eds.), Proof and Computation II. From Proof Theory and Univalent Mathematics to Program Extraction and Verification, World Scientific: Singapore 2022; K. Mainzer, The Digital and the Real World. Computational Foundations of Mathematics, Science, Technology, and Philosophy, World Scientific Singapore 2018; K. Mainzer, Artificial Intelligence. When do Machines take over, Springer: Berlin 2nd edition 2019.

(7) Title: The Transcendental Truth-Value in Some Buddhist Logic

Authors: Friend, Michele (George Washington University, Université de Lille Nord-Europe)

Abstract: In the Buddhist text: Meditation and the Concept of Insight by Kamalasila, we are told that there are five truth values: true, false, neither, both and transcendental. In modern Western logic, we are familiar with the first four. I want to discuss the fifth.

The first four apply to propositions, statements or facts. The last ties individual meta-statements to the whole doctrine or theory. In fact, it takes us beyond the exercise of careful logical argumentation to leave the exercise in its place: behind us. There are two important lessons I have learned: one is that truth-values do not have to apply only to propositions, or sets of propositions. The second is that we can use logic, or it was thought in this text that we can use logic to go beyond logic. Logic transcends itself.

To shed some light on this mysterious truth-value, let us distinguish the object-level discourse, the meta-level and the whole within a wider context.
A transcendental truth is one that occupies the object-level argumentative discourse implicitly or latently and applies in the meta-discourse as a critique of the object-level arguments. The meta-discourse is not an end. In the text, logic and careful object-level argument is about metaphysics. The questions at issue include realism and deception, our relationship to objects (so that we can let-go), our place in the universe, and our identity. The careful arguments have a purpose – to exhaust themselves. Once exhausted, an (in this respect) enlightened person can leave the object and meta-discourses behind and just be otherwise. That person is somehow released from the eros of logical reasoning, and this is the experience of coming to appreciate a transcendental truth.
Adam, Martin T. Meditation and the Concept of Insight in Kamalasita's Bhavanakramas. Commentary and translation. Ph.D. thesis, McGill University, Faculty of Religious Studies, 2003 ISBN 0-612-88405-8.