# Logic of Irreversible Reasoning

## Directed Type Theory and Its Philosophical Significance

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**SPHERE (Paris)**

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Plan :

Directed Homotopy and Directed Arithmetical Operations

Directed Type theory (DTT)

Philosophical Significance (dogmatically)

Directed Homotopy and Directed Arithmetical Operations

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# Directed Spaces, Directed Homotopy theory

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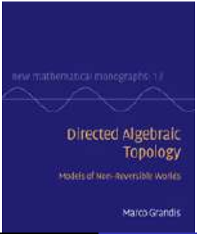

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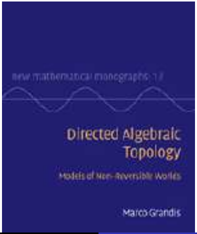
Historical source : *Formal Topology* group : Giovanni Sambin et al. since 1990s. A single person's idea ?



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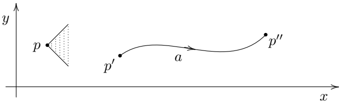
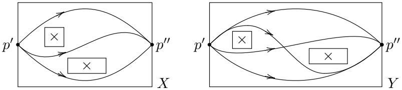



$$(x, y) \leqslant (x', y') \quad \Leftrightarrow \quad |y' - y| \leqslant x' - x.$$

Paths $p : [0, 1] \to \mathbb{R}^2$ are weakly increasing : if $t \leq t'$ in $[0, 1]$ then $p(t) \leq p(t')$ in the sense of the order defined above :

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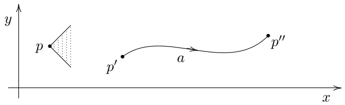


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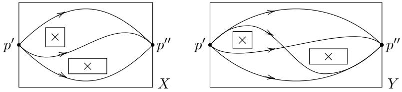


Two spaces $X, Y$ are classically of the same homotopy type. But not in the sense of *directed* homotopy.

# Elementary example of computational irreversibility (after Im. Kant 1781)

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$$\mathbb{N} \times \mathbb{N} \xrightarrow{+} \mathbb{N}$$

Map $+$ is non-invertible in the sense that there exists **no** map $+^{-1}$ of the form

$$\mathbb{N} \xrightarrow{+^{-1}} \mathbb{N} \times \mathbb{N}$$

with the properties $+^{-1} \circ + = 1_{\mathbb{N}\times\mathbb{N}}$ and $+ \circ +^{-1} = 1_{\mathbb{N}}$

Proof : map $+$ is not injective

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Informally : even if it is known that number 12 is obtained as a sum of two summands it cannot "remember" these summands.

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Remark : In the above argument we assume that operation + is *uniform* over $\mathbb{N}$ rather than specifically designed for 7 and 5. This assumption is essential. Arithmetical sums are subjects of the same schematic *rule*. Cf. Kant's argument.

# Teaching Elementary Arithmetic

Practical and pedagogical question :

What justifies the common practice of using the equality sign in 7+5=12 and the like rather than writing $7 + 5 \to 12$ ?

(A guess : the Elementary Algebra ?)

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Idem for $y = f(x)$ that should be $y \leftarrow f(x)$, etc.

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Irreversible aka *directed* features are abound in computations and elsewhere in mathematics. Elementary arithmetical operations are directed in Grandis' sense.

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Cf. the "inaddible numbers" in *Arist. Met. 13* ($\mu$)

# Historical questions

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More specifically : Where the modern common practice of using = (or its equivalents) in elementary computations comes from ? (from Algebra ?)

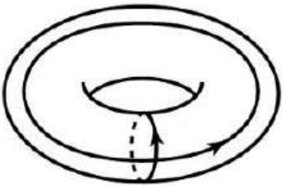
# Back to topology : the fundamental group

Recall of the notion of *fundamental group* $\pi_1(T, x_0)$ of given topological space $T$ (first introduced by Henri Poncaré in his *Analysis Situs*, 1895) :

this a group of loops with an arbitrary base point $x_0$ (where the loops are identified up to path-homotopy, which allows one to define their composition uniquely).

# Fundamental group (cntd.)

Theorem : If the given topological space $T$ is path-connected then its fundamental group does not depend on the choice of its base point.



Example : $\pi_1(S^1) \simeq \mathbb{Z}$

# Fundamental groupoids

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Remark : In the standard Homotopy theory fundamental groupoids, unlike fundamental groups, are rather useless because one cannot easily use them in (standard algebraic) computations. But HoTT makes this possible.

# Higher Homotopy groups and higher groupoids

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In case of fundamental groupoids the same move leads to the concept of *n*-groupoid (and $\infty$-groupoid) that interprets higher identity types in HoTT.

# From groups and (higher) groupoids to (higher) categories

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Higher homotopy groupoids (as in HoTT) are replaced by higher (homotopy) categories.

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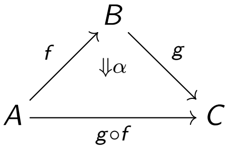
Opetopic shape (cf. $7 + 5 \to 12$)



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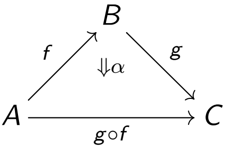
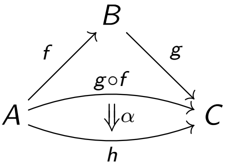



Globular shape (fits the directed Homotopy theory)



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This theoretical idea was strongly supported by the contemporary mathematical practice where the "language of categories" progressively replaced the "language of sets" (at least in a number of mainstream areas of mathematics including Algebraic Topology, Algebraic Geometry, Homological Algebra, and Functional Analysis).

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Building of Category theory (and partly also Higher Category theory) on UF is possible but it is in no way easy and natural. In UF, the Equivalence Principle holds for set-level structures (which was also a part of UF's motivation) but not for general categories (equivalent categories may have different properties).

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Hence the idea of DTT as a (version of) HoTT for directed spaces.

Directed Homotopy and Directed Arithmetical Operations

Directed Type theory (DTT)

Philosophical Significance (dogmatically)

# From the “reversible” HoTT to DTT : semantics

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# From the "reversible" HoTT to DTT : semantics

Spaces $\to$ Directed Spaces

Higher Groupoids $\to$ Higher Categories

Identity types $\to$ Hom-types (compare with Hom-sets)

# History and Biblography

The idea of DTT as an analogue of the Book HoTT that applies to directed spaces and general $\infty$-categories dates back (at least) to 2013 talk by Michael Warren in IAS Princeton (found on the youtube but not published in a paper form) : the same year that the HoTT Book has been published !

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The following presentation is based on North 2018 and North 2022 (with some critique borrowed from Altenkirch&Neumann 2024). Comparing different versions of and approaches in DTT that are presently on the market is out of the scope of this talk.

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In other words : hom-(bi)functor $C \xrightarrow{Hom(\_,\_)} Set$ is contravariant by the first variable and covariant by the second variable.

The *variance* equally matters in hom-types! This is how paths in DTT are "given a direction" (syntactically called a *variance* or *polarity*, see below. **NEW** : polarity of types is a *modality*, cf. Hugo's talk.

# Polarity syntactically (after North 2018)

Formation rules for types and terms :

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# Hom-types (after North 2018)

Hom-formation $(x \to_A y \doteq Hom_A(x, y))$ :

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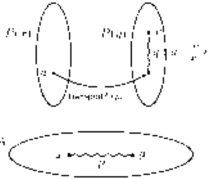
Hom-intro (notice the *refl*-term !)

$$\frac{\Gamma \vdash A : TYPE \quad \Gamma \vdash x^c : A^{core}}{\Gamma \vdash refl_x : x^{op} \rightarrow_A x} \qquad \text{(HI)}$$

# Martin-Löf's *J*-rule and transport in HoTT (for comparison)

Before I present the elimination rules for hom-types in DTT recall the elimination rule for identity types in HoTT and the notion of *transport* (along a path) that allows to interpret the rule as a form of *Indiscernibility of Identicals* :

$$\frac{\Gamma \vdash A : TYPE \quad x : A, y : A, p : x =_A y \vdash C(x,y,p) : TYPE \qquad \Gamma, x : A \vdash t(x) : C(x,x,refl(x))}{\Gamma, x : A, y : A, p : x =_A y \vdash J^t(x,y,p) : C(x,y,p)} \quad \text{(J)}$$



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Correspondingly, the $J$-rule splits into its right and left versions.

# Elimination rules for Hom-types

$$\frac{\begin{array}{c}\Gamma \vdash A : TYPE \quad \Gamma, x^c : A^{core} \vdash \Theta(x^c) : TYPE \\ \Gamma, x^c : A^{core}, y : A, f : x^{op} \rightarrow_A y, \theta : \Theta(x^c) \vdash C(f, \theta) : TYPE \\ \Gamma, x^c : A^{core}, \theta : \Theta(x^c) \vdash t(x) : C(refl_x, \theta)\end{array}}{\Gamma, x^c : A^{core}, y : A, f : x^{op} \rightarrow_A y, \theta : \Theta(x^c) \vdash J_R^t(t, f, \theta) : C(f, \theta)} \text{(RHE)}$$

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$$\frac{\begin{array}{c}\Gamma \vdash A : TYPE \quad \Gamma, y^c : A^{core} \vdash \Theta(y^c) : TYPE \\ \Gamma, x^{op} : A^{op}, y^c : A^{core}, f : x^{op} \rightarrow_A y, \theta : \Theta(y^c) \vdash C(f, \theta) : TYPE \\ \Gamma, y^c : A^{core}, \theta : \Theta(y^c) \vdash \overset{\leftarrow||}{t(y)} : C(refl_y, \theta)\end{array}}{\Gamma, y^c : A^{core}, x^{op} : A^{op}, f : x^{op} \rightarrow_A y, \theta : \Theta(y^c) \vdash J^t_L(t, f, \theta) : C(f, \theta)} \text{(LHE)}$$

# Elimination rules for Hom-types (cntd.)

Notice in RHE-LHE the presence of the intermediate structure $\Theta$, which depends on (terms of) a *core* type just like in the standard reversible HoTT , i.e. with no variance involved.

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This is the reason why Altenkirch&Neumann 2024 describe North's 2018 notion of polarity as "shallow".

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Altenkirch&Neumann argue that a *deep* notion of polarisation should also involve the polarisation of *contexts*, not only the polarisation of types over neutral (non-polarised) contexts as shown above (after North 2018). The *variance* of terms should be a primitive notion rather than an extra structure on the top of invertible types.

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Remarque that the above syntax does not admit in its semantics the general $(\infty, \infty)$-categories but only $(\infty, 1)$-categories.

Directed Homotopy and Directed Arithmetical Operations

Directed Type theory (DTT)

Philosophical Significance (dogmatically)

# (1) Categories without Structures (Rodin2011)

Category theory (CT) does not support Mathematical Structuralism as far as by a structure one understands an *invariant* under certain (group of) transformations in the spirit of Klein's *Erlangen Program* (as many people do, cf. MacLane). Or at least CT and CT-based mathematics have a *potential* to go beyond the conceptual limits of Structuralism in the usual sense of the term.

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The reason is that non-invertible morphisms admit no invariant ; the talk of "preservation of structure" in such cases is a misnomer, which can be explained by a common tendency to think about such morphisms *as if* they were invertible.

(Compare the 7+5=12 example above.)

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The Book HoTT/UF (and later Cubical TT) provided a new support for Structuralism by privileging groupoids over general categories and by saving the Equivalence Principle for set-level structures (only !).

But DTT, if equally successful, promises to restore the legacy of the category-theoretic reasoning in mathematics and explore the "irreversible worlds" (Grandis), which have been so far only very little explored (in spite of the fact that such worlds so much resemble our own).

# (2) Space, Time and Logic

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Slogan : The received notion of Logic as a set of atemporal and a-spatial fundamental principles of thought and reasoning, which has been promoted by Frege and Russell, is a Platonic prejudice that needs to be abandoned. DTT helps to take time in logic seriously

# (2) Space, Time and Logic (contd.

Objective Logic vs. Subjective Logic (Hegel/Lawvere).

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Objective Logic vs. Subjective Logic (Hegel/Lawvere).

The objectivity of common language (Aristotle, Analytic Philosophy) vs objectivity of the attained scientific knowledge about the world and about the human brain.

In the latter case spatiotemporal "geometrical" structures are indispensable.

# (3) Identity with a Sense (my talk in Lyon in January 2024 and paper to appear in *Manuscrito*)

DTT (by analogy with Book HoTT) suggests thinking of all general morphisms (of all levels) as identities. This move does not make the identity concept logically trivial and wholly counter-intuitive as one may expect.

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Some researches in DTT disagree and want to reserve the name of identity to invertible types as in the Book HoTT.

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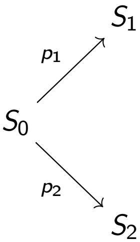
# (3) Identity with a Sense (cntd.)

The DTT-based identity *relation* is reflexive and transitive but not symmetric. It satisfies a form of *Indiscernibility of Identicals* principle (transport).

It supports the notions of identity through time and change and has a potential to account for identity talks in Biology (fission and fusion of cells, identity of plants and other organisms), Social Sciences (group identity, identity of institutions, ethnic and cultural identities), History, Geography and elsewhere where the received rigid identity concepts fail to do so.

# Theseus Ship Resolved




$S_1 = S_0$ and $S_2 = S_0$ but $S_1 \neq S_2$

(also $S_0 \neq S_1$ and $S_0 \neq S_2$)

Thanks !