Exemple d'une fonction qui possède ses dérivée partielles dans un point mais qui n'est pas différentiable dans ce point

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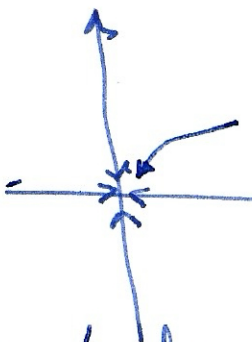
$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{si } (x,y) \neq (0,0) \\ 0 & \text{si } (x,y) = (0,0) \end{cases}$$

$$\left.\frac{\partial f}{\partial x}\right|_{y=0} = 0 \qquad \frac{\partial f}{\partial y} = 0$$

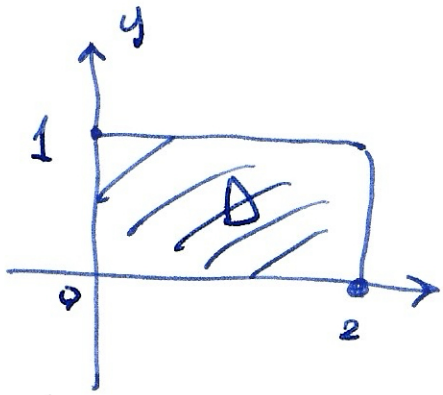


$$\lim_{\substack{(x,y)\to(0,0) \\ x=y}} \neq 0$$

$f(x,y)$ n'est pas continue, donc n'est pas différentiable ▨

Exemple d'un int. double

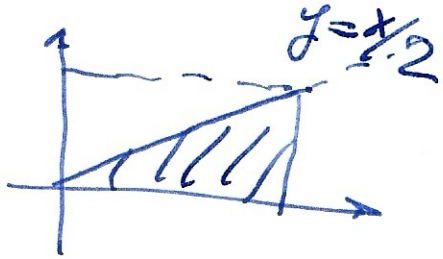
$$\iint_D xy^2\,dx\,dx =$$



$$= \int_0^1 \left(\int_0^2 xy^2\,dx \right) dy =$$

$$= \int_0^1 \left(\frac{x^2}{2} y^2 \Big|_0^2 \right) dy = \int_0^1 2y^2\,dy = \frac{2y^3}{3}\Big|_0^1 = \frac{2}{3}$$

$$\iint_{D'} xy^2\,dx\,dy =$$



$$= \int_0^2 \left(\int_0^{x/2} xy^2\,dy \right) dx = \int_0^2 \left(\frac{x}{3} y^3 \Big|_0^{x/2} \right) dx =$$

$$= \int_0^2 \frac{x^4}{24}\,dx = \frac{x^5}{5 \cdot 24}\Big|_0^2 = \frac{4}{15}$$