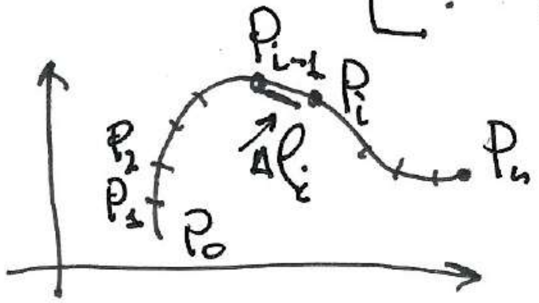
Intégrale selon une ligne courbe

$n = 2$

$$L: \quad x = x(t), \quad y = y(t) \qquad a \le t \le b$$

$$L: \quad \vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$$



Déf. $\displaystyle\int_L f(x,y)\,dl = \lim_{\max \Delta t \to 0} \sum_{i=1}^{n} f(x_i^*, y_i^*)\,\Delta l_i$

$$\Delta l_i = \sqrt{\Delta x_i^2 + \Delta y_i^2}$$

$$\int_L f(x,y)\,dl = \int_a^b f(x(t), y(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

Integrale selon une surface $\underline{n=3}$

$$S: \quad \bar r(u,v) = x(u,v)\vec{i} + y(u,v)\vec{j} + z(u,v)\vec{k}$$

$$u, v \in \underline{\underline{D}} \subseteq \mathbb{R}^3$$

$$\iint_S f(x,y,z)\,dS = \lim \sum_{i=1}^{m}\sum_{j=1}^{n} f(P_{ij}^*)\,\Delta S_{ij}$$

$$\Delta S_{ij} \approx \|\bar r_u' \times \bar r_v'\|$$

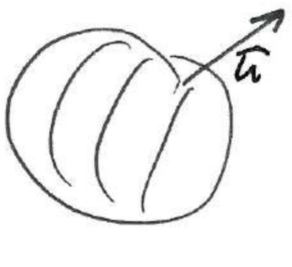
$$\bar r_u' = \frac{\partial x}{\partial u}\vec{i} + \frac{\partial y}{\partial u}\vec{j} + \frac{\partial z}{\partial u}\vec{k}$$

$$\bar r_v = \frac{\partial x}{\partial v}\vec{i} + \frac{\partial y}{\partial v}\vec{j} + \frac{\partial z}{\partial v}\vec{k}$$

$$\iint_S f(x,y,z)\,dS = \iint_D f(\bar r(u,v))\,\|\bar r_u \times \bar r_v\|\,ds$$

L'orientation d'une surface

La bande
de Möbius : une surface <u>non-orientable</u>



$\bar n$ orient.
<u>positive</u>

Si $S : \bar{r}(u,v)$ est une surface <u>orientable</u>,

alors $\bar{n} = \dfrac{r_u' \times r_v'}{\| r_u' \times r_v' \|}$ – vect. normal

$\|\bar{n}\| = 1$

<u>Déf:</u> flux de $\bar{F}$ à travers de S

$$\iint_S \bar{F} \cdot d\bar{S} = \iint_S \bar{F} \cdot \bar{n} \, dS =$$

$$= \iint_S \bar{F} \cdot \frac{\bar{r}_u' \times \bar{r}_v'}{\| \bar{r}_u' \times \bar{r}_v' \|} \, dS =$$

$$= \iint_D \left[\bar{F}(\bar{r}(u,v)) \cdot \frac{\bar{r}_u' \times \bar{r}_v'}{\| r_u' \times r_v' \|} \right] \| \bar{r}_u' \times \bar{r}_v' \| \, ds$$

$$\iint_S \bar{F} \cdot d\bar{S} = \iint_D \bar{F} \cdot (\bar{r}_u' \times \bar{r}_v') \, ds$$

Théorème de Stokes

C fermée $\displaystyle\iint_S \operatorname{rot}\vec{F}\cdot d\vec{S} = \int_C \vec{F}\cdot d\vec{r}$

$C = \partial S$

$$\iint_S \operatorname{rot}\vec{F}\cdot\vec{n}\,dS \qquad\qquad \int_C \vec{F}\cdot\vec{T}\,ds$$

Si S est plate :

$$\int_C \vec{F}\cdot d\vec{r} = \iint_S \operatorname{rot}\vec{F}\cdot d\vec{S} = \iint_S (\operatorname{rot}\vec{F})\cdot\vec{k}\,dS$$

(Th. de Green)

Théorème de divergence

$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \operatorname{div}\vec{F}\,dV$$

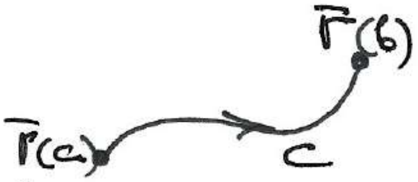
(cf. $\displaystyle\int_C \vec{F}\cdot\vec{n}\,ds = \iint_D \operatorname{div}F(x,y)\,dS$

Recap:

① Th. Fondamentale d'intégration

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

② Intégrale selon une courbe C



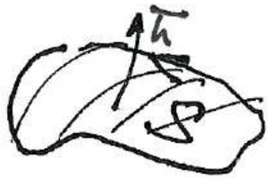
$$\int_C \bar\nabla f \cdot d\bar r = f(\bar r(b)) - f(\bar r(a))$$

③ Th. de Green

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dS = \int_C P\,dx + Q\,dy$$

④ Th. de Stokes

$$\iint_S \operatorname{rot} \bar F \cdot d\bar S = \int_C \bar F \cdot d\bar r$$



⑤ Th. de divergence

$$\iiint_E \operatorname{div} \bar F\,dV = \iint_S \bar F \cdot d\bar S$$