

Quizze 1:

October 2, 2024

1. Convert
 - (a) number 28 (written in the usual decimal positional system) into the binary system;
 - (b) number $[101101]_2$ (written in the binary system) into the usual decimal positional system;
 - (c) calculate without converting: $[101101]_2 + [11010]_2$
2. Find the number of possible 10 character passwords under the following restrictions: (Note: there are 26 lower case letters and 26 upper case letters in the alphabet.)
 - (a) All characters must be lower case letters;
 - (b) All characters must be lower case letters and distinct;
 - (c) The password can only contain the upper case letters A and B, and must contain each of these letters.
3. The four women Anne, Betsie, Charlotte and Dolores and the six men Eric, Frank, George, Harry, Ian and James are friends. Each of the women wants to marry one of the six men. In how many ways can this be done?
4. (Birthday problem.) N people are chosen from the world human population at random (which implies that chances that a chosen person is born on a particular date of the year are equal to chances that the chosen person is born on any other particular date of the year). How large should be N in order that there were more than 50 chances of 100 that at least two people of the group share their birthdays (i.e. they are born on the same date of a year)? For simplifying the problem let us ignore leap years and assume that each year comprises 365 different dates.
5. Prove by induction:
 - (a) $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$
 - (b) For all natural n number $11^n - 6$ is divisible by 5.