INTRODUCTION TO LIBERAL ARTS MATHEMATICS
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Course Description:
In the Medieval European universities the Seven Liberal Arts were classified into two unequal departments: the first one, called *trivium* comprised Grammar, Dialectics, and Rhetorics, and the second one, called *quadrivium*, which was supposed to mark an upper level of study, comprised four mathematical disciplines: arithmetic, geometry, harmonics (i.e., theory of music) and (mathematically-laden) astronomy. Even if this Medieval disciplinary structure is outdated and mostly irrelevant to today's Liberal Education, the very notion of mathematics as a liberal art remains alive and inspiring. During the last decades the were multiple attempts to develop mathematical courses specifically for Liberal Arts programs, which combined a general practical orientation with a focus on free creative mathematical theorising. The present course realises these goals in the context of today's rapidly changing technological and informational environment. The course covers issues related to the logical architecture and conceptual foundations of modern mathematics including basic elements of Combinatorics, Geometry, Calculus, Set theory, Group theory, and Topology. In addition to the mathematical contents the course includes discussions on the role and place of mathematics in the past and today's societies, which are supported by real-life examples demonstrating the power and the reach of mathematical reasoning far beyond its proper domain. In this course mathematics is treated as a human endeavour, an experimental site for human thought, which has a large impact on all spheres of the contemporary life.

When: Tuesdays and Thursdays from 18h10 to 19h30 (UTC+2: Paris and Berlin)

Where: Zoom Meeting

Office Hours: Thursdays before the class by appointment.

Organisation:

- The content is systematically exposed during the classes; the exposition includes eventual Q&As in both directions.

- Quizzes are given to students in advance as written exercises for a homework, and then reviewed in the class along with some additional exercises. The 6 quizzes included into the program represent the 6 main thematic divisions of this course.
- At the final exam each student will be asked to explain a theoretical concept studied in the class and perform a related simple exercise.
- The final score comprises the evaluation of the intermediate tests (quizzes) and work in the classroom (50 percents) and the final exam (50 percents).

Schedule (by weeks, two classes per week):

Week 1:

A) Introduction: Mathematics and the Real World. Mathematics in Society. Mathematics as a Liberal Art. Mathematics as a Science. Proofs and Refutations. Mathematics and Computers. Pure and Applies Mathematics. Mathematics and its History and Philosophy.

B) Elementary Arithmetic. Numeration Systems. Theoretical Arithmetic. The Infinity of Primes.

Week 2:

A) Combinatorics: permutations, combinations, arrangements.

B) Principle of Mathematical Induction. Proofs by Induction.

Week 3:

A) **Quiz 1:** Numeration Systems, Induction, Combinatorics.

B) Elementary Geometry. Euclid's constructions with ruler and compass. Problems and Theorems. The incommensurability of the diagonal and the side of a square. Numbers and Magnitudes. Ratios.

Week 4:

A) Geometrical Algebra, Algebraic Geometry and Cartesian Coordinates.

B) Impossible constructions in the geometry of ruler and compass, and their algebraic treatment. Regular Polygons and Regular Polyhedra.

Week 5:

A) The Axiom of Parallels and Non-Euclidean Geometries.

B) **Quiz 2:** Elementary Geometry.

Week 6:

A) The concept of continuous function and its physical meaning. Infinite series and their limits. Limit of a function.

B) Derivative and Antiderivative of a function.

Week 7:

A) Differentiation and Integration. Area under a curve.

B) **Quiz 3:** Calculus

Week 8:

A) Modern Axiomatic Method: Hilbert's Foundations of Geometry. Theories and their Models. Models of Non-Euclidean Geometry.

B) Axiomatic and genetic introduction of number concepts: Natural, Whole, Rational, Real and Complex Numbers. The Fundamental Theorem of Algebra.

Week 9:

A) Basic Set theory. Extensionality and Separation Axioms. Infinite Sets. One-to-one correspondence between sets. Cardinality of sets. Cantor Theorem and Continuum Hypothesis. Countable and non-countable sets.

B) Boolean operations on sets and propositions. Logical Connectives and Truth Tables. Boolean algebra.

Week 10:

A) Cartesian Product of sets. Relations and functions defined on sets. Equivalence and Partial Order. Equivalence classes. Whole, Rational and Real numbers revisited.

B) **Quiz 4:** Sets, Relations and Boolean Algebra.

Week 11:

A) Functions as maps. Composition of functions. Invertible and non-invertible functions. Injective and surjective maps. The concept of Category. The Category of Sets.

B) Algebraic Groups. Groups of permutations. Subgroups. Groups of geometrical transformations. Groups and Regular Polyhedra. Erlangen Program.

Week 12:

A) Isomorphic Groups. Group Homomorphisms. Mathematical Structuralism. Category of Groups.

B) **Quiz 5:** Group theory.

Week 13:

A) The concepts of Metric Space and of Topological Space. Konigsberg Bridges and Euler Characteristics of polyhedra.

B) Brouwer's Fix Point theorem and its physical implications. Fundamental groups and groupoids of topological spaces. Homotopy.

Week 14:

A) Graphs, Simplexes and Simplicial Complexes. Betti numbers. Homology. TDA: an applications of topology in the Data Analysis.

B) **Quiz 6:** Topology.

Week 15: .

A) Review and Concluding Discussion. Preparation of the Exam.

B) Final Exam.