# Axiomatic Architecture of Scientific Theories (DirectMedia 2025, in Russian)

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philomatica.org / SPHERE

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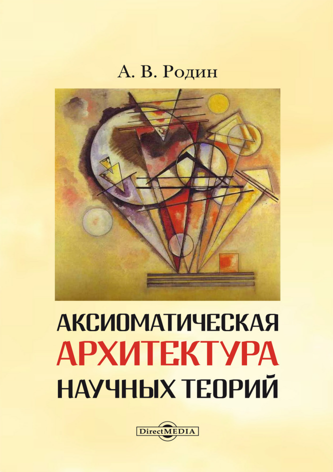
# DirectMedia 2025



# English version: a preprint

**Axiomatic Architecture of Scientific Theories**:

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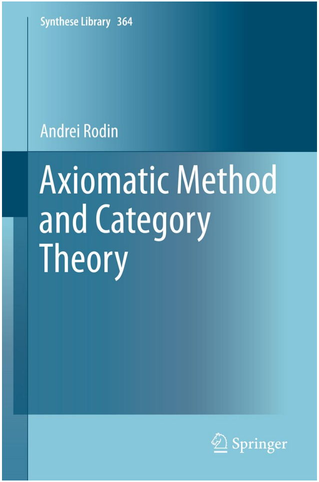
# Springer 2014



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4. Constructive Axiomatic Method.

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Example: *To produce a straight line from a point to another point.*

# Hilbert on Euclid

"Euclid does not presuppose that points or lines constitute any fixed domain of individuals. Therefore, he does not state any existence axioms either, but only construction postulates." (Hilbert & Bernays 1934)

# Ian Müller 1974: *Greek Mathematics and Greek Logic*

I know of no logic which accounts for this inference in its Euclidean formulation. One 'postulates' that a certain action is permissible and 'infers' the doing of it, i.e., does it. An obvious analogue of the procedure here is provided by the relation between rules of inference and a deduction. Rules of inference permit certain moves described in a general way. . . .

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Euclid's constructions are not inferences from his constructional postulates ; they are actions done in accord with them.

# Is Euclid's geometrical theory deductive?

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But those rules are not *logical* because some of them (Postulates) apply to non-propositional objects and none of them apply to *all* possible sentences of an appropriate logical form (as do the rules of Classical propositional logic like *modus ponens*).

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The theory of Euclid's *Elements 1-4* can be described in modern terms as a *Gentzen-style* deductive system (a lot of rules but only a few or no axioms).

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- *proof*: Every $F_\alpha$ is also closed in $T$, and hence $\{F_\alpha\}$ is a centered system of closed sets in $T$. Therefore $\cap F_\alpha \neq \emptyset$. By Theorem 1 it follows that $F$ is compact.
- *conclusion:* Any closed subset of a compact space is compact.

# Open question:

In which sense (if any) Hilbert's axiomatic theory of Euclidean geometry and the geometry of Euclid's *Elements* (Books 1-4) are *one and the same theory*?

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A tentative answer: the latter *provides for* — rather than simply "is" — a ("intended") model for the former.

But this is too week: the informal Arithmetic equally provides for such a model. The deductive structure of Euclid's geometry is wholly ignored by its Hilbertian axiomatic representation.

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Are genetic and (Hilbert-style) axiomatic methods of theory-building compatible? Yes, via HoTT

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Yet for some strange reason it passes for a model of physical axiomatics.

# Bourbaki and the Semantic View of Theories (I)

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- In the early 1950s Patrick Suppes and his collaborators rightly distinguished the Bourbaki-style axiomatic method from the Hilbert-style method and promoted applications of Bourbaki's method in *sciences* under the banner of *semantic view of theories*. The idea of *semantic view* is to identify a theory with a class of its (Tarski-style set-theoretic) *models* rather than its axioms.

# Bourbaki and the Semantic View of Theories (II)

- It triggered the development of an extensive research program (primarily in the US and the West Germany) aiming at representing a large spectrum of scientific theories using Bourbaki-style signatures that went under the name of *Structuralism*. It developed up to the early 2000th when I took part of it as an assistant to Ulises Moulines in ENS.

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- In the 1950s Suppes and his collaborators attempted to apply Bourbaki-style axiomatic representations of scientific theories on [early electronic]rcomputers, and introduce this technique into scientific research and education. Thus Suppes qualifies as a pioneer of Knowledge Representation (as a form of Artificial Intelligence).

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- Ironically, the "scientific" logically-oriented trend in the philosophy of science that aimed at narrowing the gap between science and its philosophy made that gap even wider (recall of Bunge).

# CT/MLTT/HoTT

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- The internal logic in MLTT/HoTT.

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- Types are propositions, terms are their proofs.

# Homotopical hierarchy of types (anti-cummulative) in HoTT

Definition: $S$ is a space of $h$-level $n+1$ if for all its points $x, y$ path spaces $x \ =_S \ y$ are of $h$-level $n$.

- $h$-level (-2): single point $pt$;
- $h$-level (-1): the empty space $\emptyset$ and the point $pt$: truth-values aka (mere) propositions
- $h$-level 0: sets (discrete point spaces)
- $h$-level 1: flat path groupoids : no non-contractibe surfaces
- $h$-level 2: 2-groupoids : paths and surfaces but no non-contractible volumes
- ...
-
- $h$-level $n$: $n$-groupoids
- ...
- $h$-level $\omega$: $\omega$-groupoids

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- Higher-level types are non-propositional constructions;
- Higher-level constructions justify corresponding propositions (cf. Euclid).

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- A particular derivation (formal proof) $P$ in HoTT can be materialised in form of computer program (say, in Agda). Modelling of this derivation in a specific context $C$ amounts to application $P(C)$ where $C$ is an input of this computation. The wanted model is the obtained output $M_C = P(C)$.

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In spite of its "non-sentence" character the method does not offer any new approach to modelling except the requirement that models must make true certain theoretical sentences.

# Proposed solution: empowering of the axiomatic method with genetic capabilities

In order to represent scientific theories we need a *combination* of the Hilbert-style propositional formal axiomatic method with the traditional *genetic* aka *constructive* method of concept- and theory-building, which operates with non-propositional objects.

# Constructive axiomatic method: the idea and its realisation (I)

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A tentative realisation: MLTT/HoTT

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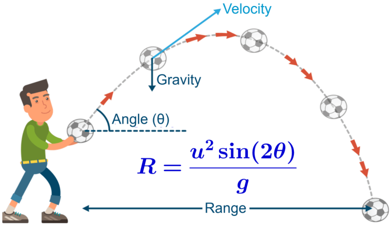
Slogan: The key ingredient of a scientific theory is neither the class of its possible syntactic (axiomatic) presentation nor the class of its (Tarski-style set-theoretic) models. It is rather the class of its *methods* justifying its claims. Vive René Descartes !

# Application of general principles to local conditions as modelling: the case of Newtonian Mechanics (I):

Newton Laws are applied in local "initial" conditions.



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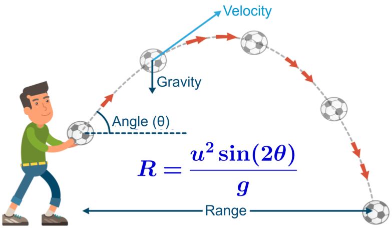

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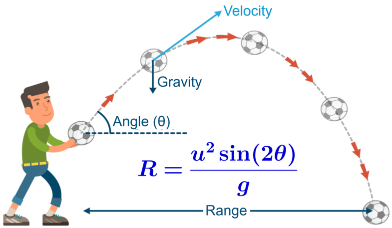
Notice non-propositional elements of this application: the trajectory of the moving projectile is modelled with an algebraic curve, viz., a parabola. This applications is more faithfully represented with a *functor* $NL \rightarrow LC$, rather than the *modus ponens*.



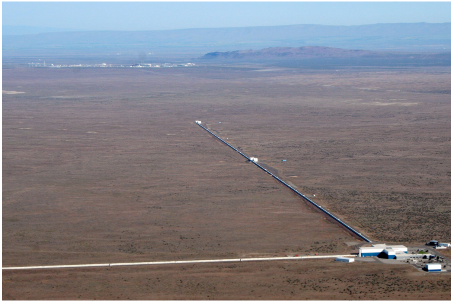
# Application of general principles to local conditions as modelling: the case of Newtonian Mechanics (II):

Newton's Laws help its users to compute a predicted trajectory of a projectile but not to satisfy some ready-made sentences. Further, the same Newtonian receipt, *mutatis mutandis*, can be used in Engineering and in the Industry.

# LIGO (2015)

The successful detection of gravitational waves in 2016, which were predicted by Einstein back in 1916 (!)

Рис.: Hanford LIGO observatory (one of the two)



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- $LIGO_{TE}$ is a taught-experiment behind LIGO (which unlike $BHC$ comprises instruments for detecting the gravitational waves). $t$ stands for the first theoretical stage of the experimental design, and $e$ stands for the engineering phase of the experimental design.

# An open Problem

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The computational obsession of our age suggests answering the above question in positive. But the historical experience with irrational algebraic reals (like $\sqrt{2}$), transcendental computable reals (like $\pi$) and "imaginary" numbers like $\sqrt{-1}$ suggests otherwise...

Thank you for your attention.

I'll be most grateful for your critical reviews.