Can Logic be possibly useful in Science?

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14 January 2026

Timeline I

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- "Galileo [1564-1642] against the philosophers" (Stillman Drake 1979). The "philosophers" in this contexts are Aristotelians who developed Logic-based "scholastic" Physics (Paris School, etc., see Pierre Duhem). Galileo criticised Aristotelian views and promoted Platonic views that gave more prominence to Mathematics rather than Logic (Mathematics as a Language of Nature).

Timeline II

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- 1920s-early1930s: Logical Positivism: *Vienna Circle* (Rudolf Carnap, Ludwig Wittgenstein and many others) as well as David Hilbert, Albert Einstein, et al.
- New Scholasticism. Jósef Maria Bochenski (1902-1995): a philosopher/priest who promoted Logic and Analytic Philosophy arguing that the Middle-Age Scholasticism is the top of human's intellectual development and the Modern Science is a decline.

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- Josef Woodger *Axiomatic Method in Biology* (1937)

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Yet for some strange reason it passes for a model of physical axiomatics.

Bourbaki and the Semantic View of Theories (I)

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- In the early 1950s Patrick Suppes and his collaborators rightly distinguished the Bourbaki-style axiomatic method from the Hilbert-style method and promoted applications of Bourbaki's method in *sciences* under the banner of *semantic view of theories*. The idea of *semantic view* is to identify a theory with a class of its (Tarski-style set-theoretic) *models* rather than its axioms.

Bourbaki and the Semantic View of Theories (II)

- It triggered the development of an extensive research program (primarily in the US and the West Germany) aiming at representing a large spectrum of scientific theories using Bourbaki-style signatures that went under the name of *Structuralism*. It developed up to the early 2000th when I took part of it as an assistant to Ulises Moulines in ENS.

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- In the 1950s Suppes and his collaborators attempted to apply Bourbaki-style axiomatic representations of scientific theories on [early electronic]rcomputers, and introduce this technique into scientific research and education. Thus Suppes qualifies as a pioneer of Knowledge Representation (as a form of Artificial Intelligence).

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- Ironically, the "scientific" logically-oriented trend in the philosophy of science that aimed at narrowing the gap between science and its philosophy made that gap even wider (recall of Bunge).

CT/MLTT/HoTT

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- The concept of *internal logic* of a category (including a topos): an analogy with that of *intrinsic geometry* of a given (Riemanean) manifold. A new relational logical framework. Lawvere axioms for the elementary topos (1970)

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- The internal logic in MLTT/HoTT.

MLTT and its Intended Proof-theoretic Semantics

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- Types are propositions, terms are their proofs.

Homotopical hierarchy of types (anti-cummulative) in HoTT

Definition: S is a space of h-level $n+1$ if for all its points x, y path spaces $x \ =_S \ y$ are of h-level n.

- h-level (-2): single point pt;
- h-level (-1): the empty space $\emptyset$ and the point pt: truth-values aka (mere) propositions
- h-level 0: sets (discrete point spaces)
- h-level 1: flat path groupoids : no non-contractibe surfaces
- h-level 2: 2-groupoids : paths and surfaces but no non-contractible volumes
- ...
-
- h-level n: n-groupoids
- ...
- h-level ω: ω-groupoids

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- Higher-level types are non-propositional constructions;
- Higher-level constructions justify corresponding propositions (cf. Euclid).

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- A particular derivation (formal proof) P in HoTT can be materialised in form of computer program (say, in Agda). Modelling of this derivation in a specific context C amounts to application $P(C)$ where C is an input of this computation. The wanted model is the obtained output $M_C = P(C)$.

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- But there is a prospective room for Logic in the (digital) Knowledge Representation.
- Today such methods are developed in the pure Mathematics (Automated Proofs). But it's time to think about extending such IT technologies to sciences beginiing with mathematically-laden sciences such as Physics.

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In spite of its “non-sentence” character the method does not offer any new approach to modelling except the requirement that models must make true certain theoretical sentences.

Proposed solution: empowering of the axiomatic method with genetic capabilities

In order to represent scientific theories we need a *combination* of the Hilbert-style propositional formal axiomatic method with the traditional *genetic* aka *constructive* method of concept- and theory-building, which operates with non-propositional objects.

Constructive axiomatic method: the idea and its realisation (I)

The idea: Methods of experimental/observational justification of scientific claims need to be included into the scope of scientific theories *and* their formal/computational representations (cf. the case of mathematical proofs).

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A tentative realisation: MLTT/HoTT

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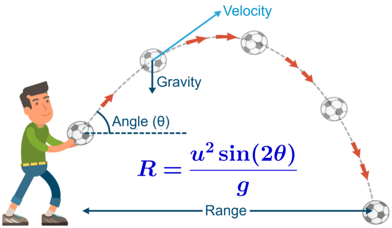
Slogan: The key ingredient of a scientific theory is neither the class of its possible syntactic (axiomatic) presentation nor the class of its (Tarski-style set-theoretic) models. It is rather the class of its *methods* justifying its claims. Vive René Descartes !

Application of general principles to local conditions as modelling: the case of Newtonian Mechanics (I):

Newton Laws are applied in local “initial” conditions.



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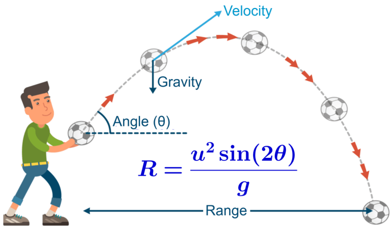

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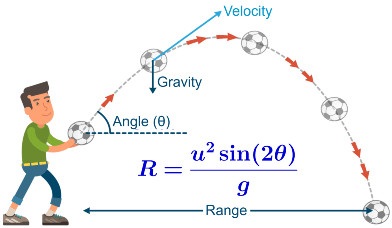
Notice non-propositional elements of this application: the trajectory of the moving projectile is modelled with an algebraic curve, viz., a parabola. This applications is more faithfully represented with a *functor* $NL \to LC$, rather than the *modus ponens*.



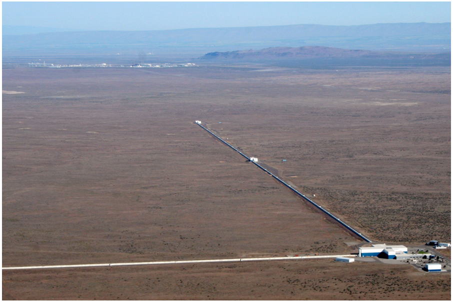
Application of general principles to local conditions as modelling: the case of Newtonian Mechanics (II):

Newton's Laws help its users to compute a predicted trajectory of a projectile but not to satisfy some ready-made sentences. Further, the same Newtonian receipt, *mutatis mutandis*, can be used in Engineering and in the Industry.

LIGO (2015)

The successful detection of gravitational waves in 2016, which were predicted by Einstein back in 1916 (!)

Рис.: Hanford LIGO observatory (one of the two)



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- GR is the theory of General Relativity;
- BHC is the GR-based physical model of colliding black holes, which predicts the emission of gravitational waves; m stands for modelling.
- $LIGO_{TE}$ is a taught-experiment behind LIGO (which unlike BHC comprises instruments for detecting the gravitational waves). t stands for the first theoretical stage of the experimental design, and e stands for the engineering phase of the experimental design.

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The computational obsession of our age suggests answering the above question in positive. But the historical experience with irrational algebraic reals (like $\sqrt{2}$), transcendental computable reals (like π) and "imaginary" numbers like $\sqrt{-1}$ suggests otherwise...

Thank you for your attention.

I'll be most grateful for your critical reviews.