Долгая история формализации и деформализации математики

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Motivation: Big Proof series: 2017, 2019, 2025

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Big proof: formalizing mathematics at scale

BPRW03

9 June 2025 to 13 June 2025

Workshop theme

The 2025 Big Proof workshop is a follow-up to the successful 2017 Big Proof programme at the INI and the 2019 follow-up workshop at ICMS. Since these workshops, there has been an explosion of work in the large-scale formalization of mathematics with spinoff activity targeting mathematical models in other fields. The 2025 Workshop is an opportunity to exchange experiences and ideas and craft a forward-looking research roadmap.

The workshop will focus on pragmatic foundations, scalable proof automation, tradeoffs between expressiveness and automation, interchange formats, indexable digital libraries, the role of machine learning in proof, social aspects of digital mathematics, and broader applications of proof technology. We hope to build on the enthusiastic response to prior Big Proof events to plan and launch major initiatives around the large-scale formalization of mathematical knowledge.

https://www.newton.ac.uk/event/bprw03/

Euclid: Pythagorean Theorem 1, ca 300 BC

μζ΄.

Ἐν τοῖς ὀρθογωνίοις τριγώνοις τὸ ἀπὸ τῆς τὴν ὀρθὴν γωνίαν ὑποτεινούσης πλευρᾶς τετράγωνον ἴσον ἐστὶ τοῖς ἀπὸ τῶν τὴν ὀρθὴν γωνίαν περιεχουσῶν πλευρῶν τετραγώνοις.

Ἔστω τρίγωνον ὀρθογώνιον τὸ ΑΒΓ ὀρθὴν ἔχον τὴν ὑπὸ ΒΑΓ γωνίαν· λέγω, ὅτι τὸ ἀπὸ τῆς ΒΓ τετράγωνον ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΒΑ, ΑΓ τετραγώνοις.

Ἀναγεγράφθω γὰρ ἀπὸ μὲν τῆς ΒΓ τετράγωνον τὸ ΒΔΕΓ, ἀπὸ δὲ τῶν ΒΑ, ΑΓ τὰ ΗΒ, ΘΓ, καὶ διὰ τοῦ Α ὁποτέρᾳ τῶν ΒΔ, ΓΕ παράλληλος ἤχθω ἡ ΑΛ· καὶ ἐπεζεύχθωσαν αἱ ΑΔ, ΖΓ. καὶ ἐπεὶ ὀρθή ἐστιν ἑκατέρα τῶν ὑπὸ ΒΑΓ, ΒΑΗ γωνιῶν, πρὸς δή τινι εὐθείᾳ τῇ ΒΑ καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ Α δύο εὐθεῖαι αἱ ΑΓ, ΑΗ μὴ ἐπὶ τὰ αὐτὰ μέρη κείμεναι τὰς ἐφεξῆς γωνίας δυσὶν ὀρθαῖς ἴσας ποιοῦσιν· ἐπ᾿ εὐθείας ἄρα ἐστὶν ἡ ΓΑ τῇ ΑΗ. διὰ τὰ αὐτὰ δὴ καὶ ἡ ΒΑ τῇ ΑΘ ἐστιν ἐπ᾿ εὐθείας. καὶ ἐπεὶ ἴση ἐστὶν ἡ ὑπὸ ΔΒΓ γωνία τῇ ὑπὸ ΖΒΑ· ὀρθὴ γὰρ ἑκατέρα· κοινὴ προσκείσθω ἡ ὑπὸ ΑΒΓ· ὅλη ἄρα ἡ ὑπὸ ΔΒΑ ὅλῃ τῇ ὑπὸ ΖΒΓ ἐστιν ἴση. καὶ ἐπεὶ ἴση ἐστὶν ἡ μὲν ΔΒ τῇ ΒΓ, ἡ δὲ ΖΒ τῇ ΒΑ, δύο δὴ αἱ ΔΒ, ΒΑ δύο ταῖς ΖΒ, ΒΓ ἴσαι εἰσὶν ἑκατέρα ἑκατέρᾳ· καὶ γωνία

Proposition 47

In a right-angled triangle, the square on the side subtending the right-angle is equal to the (sum of the) squares on the sides surrounding the right-angle.

Let ABC be a right-angled triangle having the right-angle BAC. I say that the square on BC is equal to the (sum of the) squares on BA and AC.

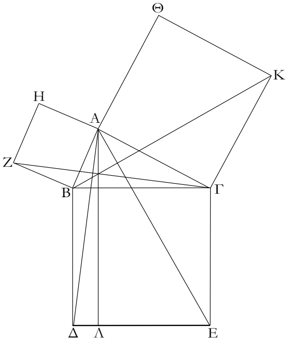
For let the square $BDEC$ have been described on BC, and (the squares) GB and HC on AB and AC (respectively) [Prop. 1.46]. And let AL have been drawn through point A parallel to either of BD or CE [Prop. 1.31]. And let AD and FC have been joined. And since angles BAC and BAG are each right-angles, then two straight-lines AC and AG, not lying on the same side, make the (sum of the) adjacent angles equal to two right-angles at the same point A on some straight-line BA . Thus, CA is straight-on to AG [Prop. 1.14]. So, for the same (reasons), BA is also straight-on to AH. And since angle DBC is equal to FBA, for (they are) both right-angles, let ABC have been added to both. Thus, the whole (angle) DBA is equal to the whole (angle) FBC. And since DB is equal to BC, and FB to BA,

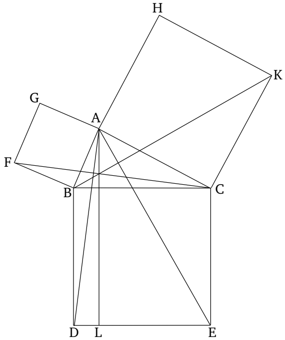
Euclid: Pythagorean Theorem 2, ca 300 BC

ἡ ὑπὸ ΔΒΑ γωνίᾳ τῇ ὑπὸ ΖΒΓ ἴση· βάσις ἄρα ἡ ΑΔ βάσει τῇ ΖΓ [ἐστιν] ἴση, καὶ τὸ ΑΒΔ τρίγωνον τῷ ΖΒΓ τριγώνῳ ἐστὶν ἴσον· καί [ἐστι] τοῦ μὲν ΑΒΔ τριγώνου διπλάσιον τὸ ΒΛ παραλληλόγραμμον· βάσιν τε γὰρ τὴν αὐτὴν ἔχουσι τὴν ΒΔ καὶ ἐν ταῖς αὐταῖς εἰσι παραλλήλοις ταῖς ΒΔ, ΑΛ· τοῦ δὲ ΖΒΓ τριγώνου διπλάσιον τὸ ΗΒ τετράγωνον· βάσιν τε γὰρ πάλιν τὴν αὐτὴν ἔχουσι τὴν ΖΒ καὶ ἐν ταῖς αὐταῖς εἰσι παραλλήλοις ταῖς ΖΒ, ΗΓ. [τὰ δὲ τῶν ἴσων διπλάσια ἴσα ἀλλήλοις ἐστίν·] ἴσον ἄρα ἐστὶ καὶ τὸ ΒΛ παραλληλόγραμμον τῷ ΗΒ τετραγώνῳ. ὁμοίως δὴ ἐπιζευγνυμένων τῶν ΑΕ, ΒΚ δειχθήσεται καὶ τὸ ΓΛ παραλληλόγραμμον ἴσον τῷ ΘΓ τετραγώνῳ· ὅλον ἄρα τὸ ΒΔΕΓ τετράγωνον δυσὶ τοῖς ΗΒ, ΘΓ τετραγώνοις ἴσον ἐστίν. καί ἐστι τὸ μὲν ΒΔΕΓ τετράγωνον ἀπὸ τῆς ΒΓ ἀναγραφέν, τὰ δὲ ΗΒ, ΘΓ ἀπὸ τῶν ΒΑ, ΑΓ. τὸ ἄρα ἀπὸ τῆς ΒΓ πλευρᾶς τετράγωνον ἴσον ἐστὶ τοῖς ἀπὸ τῶν ΒΑ, ΑΓ πλευρῶν τετραγώνοις.

the two (straight-lines) DB, BA are equal to the two (straight-lines) CB, BF,[†] respectively. And angle DBA (is) equal to angle FBC. Thus, the base AD [is] equal to the base FC, and the triangle ABD is equal to the triangle FBC [Prop. 1.4]. And parallelogram BL [is] double (the area) of triangle ABD. For they have the same base, BD, and are between the same parallels, BD and AL [Prop. 1.41]. And parallelogram GB is double (the area) of triangle FBC. For again they have the same base, FB, and are between the same parallels, FB and GC [Prop. 1.41]. [And the doubles of equal things are equal to one another.][‡] Thus, the parallelogram BL is also equal to the square GB. So, similarly, AE and BK being joined, the parallelogram CL can be shown (to be) equal to the square HC. Thus, the whole square $BDEC$ is equal to the (sum of the) two squares GB and HC. And the square $BDEC$ is described on BC, and the (squares) GB and HC on BA and AC (respectively). Thus, the square on the side BC is equal to the (sum of the) squares on the sides BA and AC.

Euclid: Pythagorean Theorem 3, ca 300 BC





Ἐν ἄρα τοῖς ὀρθογωνίοις τριγώνοις τὸ ἀπὸ τῆς τὴν ὀρθὴν γωνίαν ὑποτεινούσης πλευρᾶς τετράγωνον ἴσον ἐστὶ τοῖς ἀπὸ τῶν τὴν ὀρθὴν [γωνίαν] περιεχουσῶν πλευρῶν τετραγώνοις· ὅπερ ἔδει δεῖξαι.

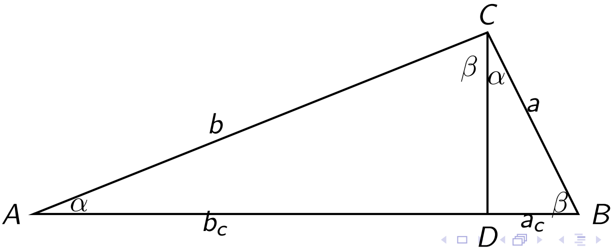
Thus, in a right-angled triangle, the square on the side subtending the right-angle is equal to the (sum of the) squares on the sides surrounding the right-[angle]. (Which is) the very thing it was required to show.

Pythagorean Theorem in a modern textbook

Theorem: Let a, b, c are sides of rectangular triangle and c its hypothenuse. Then $a^2 + b^2 = c^2$.

Proof: Let a_c, b_c be projections of a, b on the hypothenuse c. Then, by the equality of angles,
$\triangle ABC \sim \triangle ACD \sim \triangle DCB$. It follows that

$$\frac{a}{c} = \frac{a_c}{a}, \quad \frac{b}{c} = \frac{b_c}{b} \Rightarrow a^2 = a_c c, \quad b^2 = b_c c \Rightarrow a^2 + b^2 = c^2 \blacksquare$$



Pythagorean Theorem in a modern textbook

The above modern textbook proof of the Pythagorean theorem involves ideas and notation first systematically introduced by René Descartes in 1637.

It does not essentially use any mathematical technique or knowledge obtained later on.

Al-Khwarizmi, Algebra, ca 820 CE

(12)

the number of the roots must be halved. And know, that, when in a question belonging to this case you have halved the number of the roots and multiplied the moiety by itself, if the product be less than the number of dirhems connected with the square, then the instance is impossible;* but if the product be equal to
(8) the dirhems by themselves, then the root of the square is equal to the moiety of the roots alone, without either addition or subtraction.

In every instance where you have two squares, or more or less, reduce them to one entire square,† as I have explained under the first case.

Roots and Numbers are equal to Squares;‡ for instance, "three roots and four of simple numbers are equal to a square." Solution: Halve the roots; the moiety is one and a half. Multiply this by itself; the product is two and a quarter. Add this to the four; the sum is

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* If in an equation, of the form $x^2+a=bx$, $(\frac{b}{2})^2 \angle a$, the case supposed in the equation cannot happen. If $(\frac{b}{2})^2=a$, then $x=\frac{b}{2}$

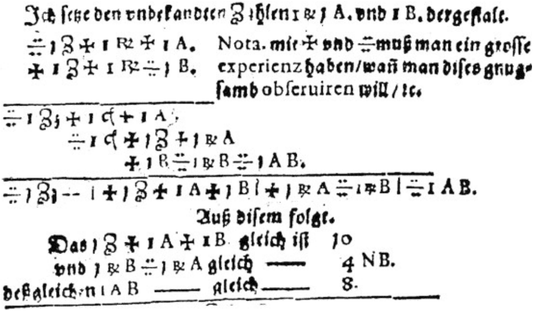
† $cx^2+a=bx$ is to be reduced to $x^2+\frac{a}{c}=\frac{b}{c}x$

‡ 3d case $cx^2=bx+a$

Example $x^2=3x+4$

$$x^2=\sqrt{[(\tfrac{3}{2})^2+4]}+\tfrac{3}{2}$$
$$=\sqrt{(1\tfrac{1}{4})^2+4}+1\tfrac{1}{2}$$
$$=\sqrt{2\tfrac{1}{4}+4}+1\tfrac{1}{2}$$
$$=\sqrt{6\tfrac{1}{4}}+1\tfrac{1}{2}$$
$$=2\tfrac{1}{2}+1\tfrac{1}{2}=4$$

Johann Faulhaber, *Miracula Arithmetica*, 1622


Ich setze den unbekandten Zahlen 1 ℞ 1 A. und 1 B. dergestalt.

÷ 1 ʒ + 1 ℞ + 1 A. Nota. mit + und ÷ muß man ein grosse
+ 1 ʒ + 1 ℞ ÷ 1 B. experienz haben/wann man dises genau-
samb obseruiren will/ etc.

÷ 1 ʒ + 1 ᶜ + 1 A.
÷ 1 ᶜ + 1 ʒ + 1 ℞ A
+ 1 B ÷ 1 ℞ B ÷ 1 A B.

÷ 1 ʒ -- | + 1 ʒ + 1 A + 1 B | + 1 ℞ A ÷ 1 ℞ B | ÷ 1 A B.

Auß disem folgt.

Das 1 ʒ + 1 A + 1 B gleich ist 10
und 1 ℞ B ÷ 1 ℞ A gleich —— 4 NB.
deßgleichen 1 A B —— gleich —— 8.

Descartes *La géométrie* 1637

GEOMETRIA,
à
RENATO DES CARTES
Anno 1637 Gallicè edita; poſtea autem
Unà cum Notis
FLORIMONDI DE BEAUNE,
In Curia Bleſenſi Conſiliarii Regii, Gallicè conſcriptis in
Latinam linguam verſa, & Commentariis illuſtrata,
Operâ atque ſtudio
FRANCISCI à SCHOOTEN,
in Acad. Lugd. Batava Matheſeos Profeſſoris.
Nunc demum ab eodem diligenter recognita, locupletioribus Commentariis inſtructa, multiſque egregiis acceſſionibus, tam ad ulteriorem explicationem, quam ad ampliandam hujus Geometriæ excellentiam facientibus, exornata,
Quorum omnium Catalogum pagina verſa exhibet.

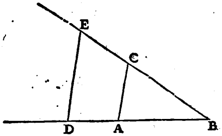
INDEFESSUS AGENDO

AMSTELODAMI,
Ex Typographia BLAVIANA, MDCLXXXIII.
Sumptibus Societatis.

Descartes *La géométrie* 1637

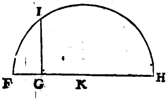
Quomodo Geometrice fiat. reſve mediæ proportionales, quod idem eſt, quod radicis Quadratæ, aut Cubicæ, &c. extractio. Neque enim hoſce Arithmetices terminos, ut faciliùs intelligi poſſim, in Geometriam introducere verebor.

Multiplicatio, Sit, exempli gratiâ, A B unitas, oporteatque multiplicare B D per B C: jungo puncta A & C, ductâque D E parallelâ A C, erit B E productum hujus multiplicationis.



Diviſio, Vel ſi dividenda ſit B E per B D, junctis punctis E & D, duco A C parallelam ipſi D E, eritque B C quotiens hujus Diviſionis.

Extractio radicis Quadratæ. Vel denique ſi ex G H extrahere oporteat radicem Quadratam, adjungo ipſi in directum lineam rectam F G, quæ unitas eſt; diviſâque F H bifariam in puncto K, centro K intervallo F K ſeu K H deſcribo circulum. quo facto, erit G I, quæ ex puncto G perpendicularis ducitur ſuper F H uſque ad I, radix quæſita.



Leibniz, *De arte characteristica ad perficiendas scientias ratione nitentes*, 1688 (I)

Si daretur vel lingu a quaedam exacta (qualem quidam Adamicam vocant) vel saltem genus Scripturae vere philosophicae, qua notiones revocarentur ad Alphabet um quoddam cogitatio num humanarum, omnia quae ex datis ratione assequi licet, inveniri possent, quodam genere calculi, perinde ac resolvuntur problemata Arithmeticae aut Geometriae.

Если бы нам удалось найти некоторый точный язык (который некоторые называют *адамическим*) или что-то в роде истинно философского способа письма основанного на алфавите человеческого мышления, то все логические следствия данных нам предпосылок можно было бы получить с помощью вычислений подобно тому, как это делается при решении арифметических и геометрических задач.

Leibniz, *De arte characteristica ad perficiendas scientias ratione nitentes*, 1688 (II)

[I]ta sentio nunquam temere controversias finiri neque sectis silentium imponi posse, nisi a ratiocinationibus complicatis ad calculos simplices, a vocabulis vagae incertaeque significationis, ad characteres determinatos revocemur.

Поэтому я думаю что наши споры никогда не закончатся, пока мы не научимся сводить сложные рассуждения к простым вычислениям и представлять наши туманные речи точными символами.

Leibniz, *De arte characteristica ad perficiendas scientias ratione nitentes*, 1688 (III)

Quae vero facti sunt, et a fortuna vel casu pendent eatenus ad artem inveniendi non pertinere manifestum est.

Однако ясно, что факты зависящие от судьбы или случая не могут стать известными с помощью такой изобретательной техники.

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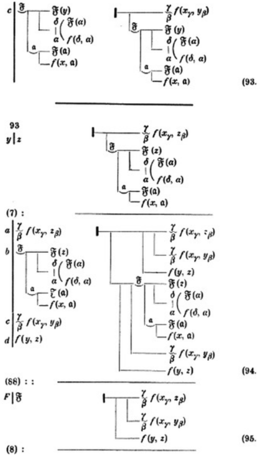
Commentary: Leibniz does not assume completeness of his projected logical calculus. But he might be nevertheless surprised to learn that some arithmetical truths are contingent.

Frege *Begrieffsschrift* (1879)



Hilbert 1927

"С помощью этого нового способа обоснования математики, который можно назвать теорией доказательства, я хочу раз и навсегда избавить от всех вопросов оснований математики в той форме, в которой эти вопросы сегодня ставятся. Для этой цели я превращаю каждое математическое утверждение в формулу, которая может быть конкретно построена и строго выведена. .... Я верю в то, что с помощью моей теории доказательств я смогу достичь поставленной цели."

Metamathematics

But as a matter of historical fact Hilbert and his followers using formalisation pursued primarily *metamathematical* goals: they proved important (meta)theorems concerning theoretical *provability* and similar general issues; they did not use formal approaches either in solving open problems or in teaching.

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Formal Set theory and a mathematical theory *about* ZF(C) including its model theory. Set theoretic foundations of mathematics inherit the same feature: they operate in the ideal modality "in principle" but not as a practical guide of how to prove mathematical theorems.

Alienation between theoreticians and practitioners

This created a profound alienation between the core mathematical community and the community of logicians/mathematicians/philosophers working on *foundations* of mathematics. This concerns many ATP enthusiasts and developers (Kevin Buzzard).

Lawvere and Rosebrugh 2003

"Основания науки объясняют ее существенные общие характеристики, составные части и операции, а также происхождение данной науки и законы ее развития. Задача оснований науки состоит в том, чтобы дать руководство к изучению, использованию и дальнейшему усовершенствованию этой науки. "Чистые" основания, которые забывают об этой задаче и уходят в оторванные от научной практики спекуляции, очевидно, не являются основаниями."

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“Theoretical” and “practical” foundations/philosophy of maths (Paolo Mancosu)

Giuseppe Peano, *Formulario* 1895

Formulario: How an ultimate formalisation looks like in practice.

III.

§ 1.

$a, b, c, k \,\varepsilon\, \mathrm{n} \,.\, \mathrm{Ɔ} :$

1. $0 \,\varepsilon\, \mathrm{n}\, a \,.$
2. $a \,\varepsilon\, \mathrm{n} \times 1 \,.$
3. $a \,\varepsilon\, \mathrm{n}\, a \,.$
4. $ab \,\varepsilon\, \mathrm{n}\, a \,.$
5. $a \,\varepsilon\, \mathrm{n}\, b \,.\, b \,\varepsilon\, \mathrm{n}\, a \,.=.\, a = \pm b \,.$
6. $a \,\varepsilon\, \mathrm{n}\, b \,.\, b \,\varepsilon\, \mathrm{n}\, c \,.\mathrm{Ɔ}.\, a \,\varepsilon\, \mathrm{n}\, c \,.$
7. $a, b \,\varepsilon\, \mathrm{n}\, c \,.\mathrm{Ɔ}.\, a + b, a - b \,\varepsilon\, \mathrm{n}\, c \,.$
8. $a \,\varepsilon\, \mathrm{n}\, b \,.\mathrm{Ɔ}.\, ac \,\varepsilon\, \mathrm{n}\, bc \,.$
9. » $.\mathrm{Ɔ}.\, ac \,\varepsilon\, \mathrm{n}\, b \,.$
10. $a \,\varepsilon\, b + \mathrm{n}\, k \,.\, b \,\varepsilon\, c + \mathrm{n}\, k \,.\mathrm{Ɔ}.\, a \,\varepsilon\, c + \mathrm{n}\, k \,.$
11. » $.\, a' \,\varepsilon\, b' + \mathrm{n}\, k \,.\mathrm{Ɔ}.\, a + a' \,\varepsilon\, b + b' + \mathrm{n}\, k \,.$
12. » $.\mathrm{Ɔ}.\, ca \,\varepsilon\, cb + \mathrm{n}\, k \,.$
13. » $.\, a' \,\varepsilon\, b' + \mathrm{n}\, k \,.\mathrm{Ɔ}.\, aa' = bb' + \mathrm{n}\, k \,.$
14. » $.\mathrm{Ɔ}.\, a^m \,\varepsilon\, b^m + \mathrm{n}\, k \,.$
15. $ca \,\varepsilon\, cb + \mathrm{n}\, ck \,.\mathrm{Ɔ}.\, a \,\varepsilon\, b + \mathrm{n}\, k \,.$

Giuseppe Peano, *Formulario* 1901

Deformalisation of *Formulario*: a historical precursor of MALINCA project:

✱ 4. **Propositions primitives**

·0 $N_0 \; \varepsilon \; \mathrm{Cls}$ Pp ·1 $0 \, \varepsilon \, N_0$ Pp

·2 $a \varepsilon N_0 \,.\!\supset\!.\, a{+} \, \varepsilon \, N_0$ Pp

·3 $s \varepsilon \mathrm{Cls} \,.\, 0 \varepsilon s : x \varepsilon s \,.\!\supset_x\!.\, x{+} \, \varepsilon s :\supset.\, N_0 \supset s$ Pp

{ P·3 = Induct = « loi d'induction » {

{ Pascal a.1654 t.3 p.298 :

« Premier lemme ... cette proposition se rencontre dans la seconde base... Deuxième ... si cette proposition se trouve dans une base quelconque, elle se trouvera nécessairement dans la suivante.

D'où il se voit qu'elle est nécessairement dans toutes les bases. » {

Les idées primitives sont déterminées par les propositions primitives que nous venons d'énoncer et par les P6·2 P8·4, desquelles découlent toutes les P de l'Arithmétique.

Dans la lecture des propositions il convient de se rapprocher autant que possible du langage ordinaire. On lira les P4 p. ex. comme il suit:

·0 « N_0 est une classe » ·1 « à laquelle appartient 0 »

·2 « Tout nombre est suivi par un nombre. »

·3 « Soit *s* une classe; supposons que 0 appartienne à cette classe; et que toutes les fois qu'un individu appartient à cette classe, son suivant y appartienne aussi; alors tous les nombres appartiennent à cette classe. »

Lebombo bone, the estimated age is 42-43 Kyears

30 notches found on this bone may correspond to the Lunar month.

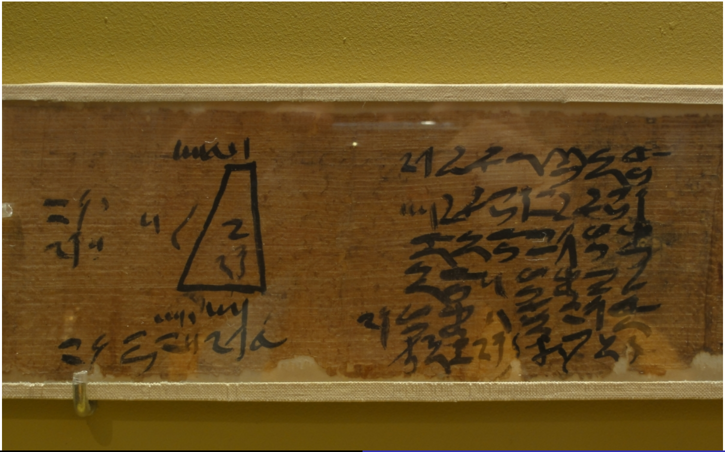
Lebombo bone, the estimated age is 42-43 Kyears

30 notches found on this bone may correspond to the Lunar month. Human *mathematical* scripts (or samplings) historically (by far) predates *writing* (in the usual sense of graphical representation of spoken language).

Clay accounting tokens from Tello (anc. Girsu), excavations Genouillac 1930-31, ca. 3500-2900 BC.

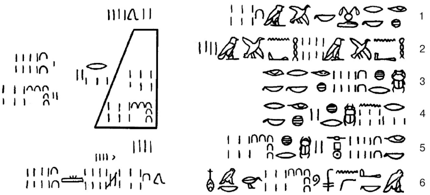
Moscow (Golenischev) papyrus, ca. 2000 BC

Combining a mathematical script with writing (a text in natural language) as a form of (meaning-giving?) *deformalisation*.



Moscow (Golenischev) papyrus, ca. 2000 BC

If you are told [problem]: a truncated pyramid of 6 for the vertical hight by 4 on the base by 2 on the top. [Solution:] You to square this 4; result 16. You are to double 4; result 8. You are to square this 2; result 4. You are to add the 16 and the 18 and the 4; result 28. You are to take $\frac{1}{3}$ of 6; result 2. You are to take 28 twice; result 56. See, it is of 56. You found it right.



Moscow (Golenischev) papyrus: commentary

Text in the right column is written in Ancient Egyptian language using hieratic writing along with standard arithmetical symbols. It parallels the formal computation presented in the left column.

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Why the author of the document opted for supporting the formal computation with a text in (the hieratically written) Egyptian?

Theoretical Mathematics: back to Euclid

It appears that the *theoretical* mathematics concerned with proof, reasoning and understanding — as we know it, in particular, via Euclid's *Elements* — would be impossible without using the natural language in a systematic way.

1. Formalisation from Euclid to Coquand
2. Deformalisation from the Stone Age to Euclid
3. **Automated Theorem Proving**
4. Where are we, and where are we going?

Timeline

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- 1956: Dartmouth Workshop (*Dartmouth Summer Research Project on Artificial Intelligence* organised by Claude Shannon, John MacCarthy et al.): the official birth of AI.

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- 1968: Versaille Symposium (*Symposiusm on Automatic Demonstration* organised by M. Laudet et al.). Public presentation of *AUTOMATH* by Nicolaas de Bruijn

MacCarthy 1961 *Computer programs for checking mathematical proofs*

Checking mathematical proofs is potentially one of the most interesting and useful applications of automatic computers. Computers can check not only the proofs of new mathematical theorems but also proofs that complex engineering systems and computer programs meet their specifications.

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Proofs to be checked by computer may be briefer and easier to write than the informal proofs acceptable to mathematicians. This is because the computer can be asked to do much more work to check each step than a human is willing to do, and this permits longer and fewer steps. [. . .]

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The combination of proof-checking techniques with proof-finding heuristics will permit mathematicians to try out ideas for proofs

Viktor M. Glushkov (Виктор Михайлович Глушков) 1970: the concept of proof assistant

Since 1962 the director of newly founded *Institute of Cybernetics* (today of Ukrainian National Academy of Science) in Kyiv.

1970: the concept of proof assistant

"Что касается, в частности, автоматизации доказательства теорем, то центр тяжести работы в этой области должен быть смещен от построения "универсальных" доказывающих программ в сторону создания систем автоматизации программирования и операционных систем, позволяющих, в случае необходимости, быстро программировать поиск доказательства даже одной-единственной трудной теоремы и способных, если потребуется, работать в истинном масштабе времени с математиком, доказывающим эту теорему. На таком и только на таком пути можно за разумное время добиться ощутимых практических результатов."
("Некоторые проблемы теории автоматов и искусственного интеллекта", *Кибернетика* 1970, номер 2 : Собр. соч. том 2, стр. 203)

Timeline (continued)

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- since mid-1960s –?: AUTOMATH (de Bruijn)

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Timeline (continued)

New theoretical development: Type theory with dependent types (Per Martin-Löf)

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Timeline (continued)

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- 1984: NURPL (Robert Lee Constable)
- 1989: COQ (Thierry Coquand)
- 1999: AGDA (Catarina Coquand)
- since 2013: LEAN 1–4 (originally Leonardo de Moura, presently Microsoft Research and a large Lean community)

LEAN

LEAN THEOREM PROVER

https://leanprover-community.github.io/index.html

LEAN

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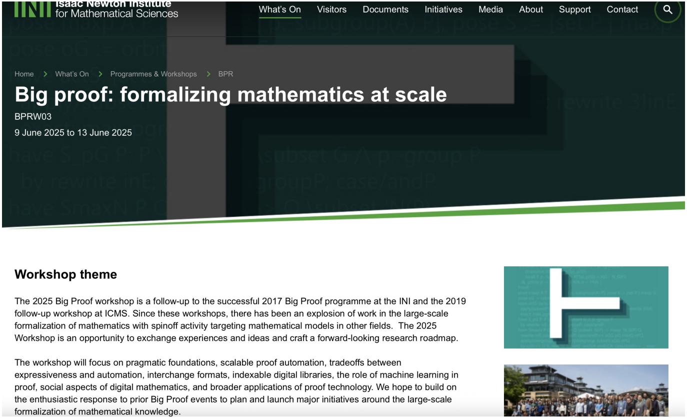
https://github.com/UniMath/UniMath (Coq)

Big Proof workshop series: 2017, 2019, 2025



https://www.newton.ac.uk/event/bprw03/

MALINCA project

Construire un pont linguistique entre le mathématicien et la machine, tel est le sous-titre de MALINCA. Ce projet est soutenu par une bourse du Conseil européen de la recherche ERC Synergy qui, contrairement aux financements ERC classiques, est destinée à toute une équipe de scientifiques.rMALINCA vise à élargir l'usage des mathématiques vérifiées par ordinateur, en faisant notamment fonctionner divers outils de vérification et de preuves mathématiques avec le langage naturel. Une démonstration mathématique pourrait alors être directement et automatiquement vérifiée par ordinateur, sans avoir à être traduite en code. Cela aiderait à gérer le flux grandissant de publications en mathématiques qui sortent chaque mois.

https://www.ins2i.cnrs.fr/fr/cnrsinfo/
malinca-ou-comment-verifier-automatiquement-les-theoremes-s

Adamic Language

The idea of *Adamic Language* (or *Adamic Script*) in the form of logical calculus which directly reflects basic ontological structures of our universe and thus enables us to know things in this universe and to reason about them correctly is a anti-scientific myth, which underlies a good deal of (so called *Analytic* Philosophy in the 20th century.

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Leibniz himself referred to the notion of *Adamic Language* rather ironically, but two centuries later Bertrand Russell took the idea of *Adamic Script* quite seriously.

Neo-Scholasticism

In the course of the 20th century a large part of the Analytic Philosophy developed into a new form of dogmatic medieval-style Scholasticism where Aristotle's syllogisms were replaced by the elements of the Classical First-Order Logic.

Józef Maria Bocheński (1902-1995)

Józef Bochenski's fully embraced and emphasised the conceptual ties of Analytic Philosophy to the Medieval Scholasticism. He believed that his contemporary Analytic Scholasticism was an appropriate antidote against his contemporary Marxism and the related doctrine of *Dialectical Materialism* promoted by Kremlin ideologues and their heirs in other countries of the Communist Block (including Poland where Bocheńsky was born).

An exit strategy

The recent booming development of new formal calculi in Mathematical Logic and Computer Science, and the implementation of many of those calculi in the form of executable programs, undermines the monopoly of the Classical FOL in its pretended role of *the* language of rational argument and thought (i.e., of the *Adamic Script*) and calls for a revision of our current ideas about formal calculi and their epistemic roles.

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What we need today is not yet another replacement of Classical FOL by some more fashionable formal calculus but rather a reconsidering of the relationships between formal calculi, natural languages and other related symbolic tools used in our scientific, educational, and cultural practices.

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- What are criteria of the adequacy of a formalisation of some given informal content?
- (Dually:) What are criteria of the adequacy of a deformalisation (aka interpretation) of some given symbolic content?
- What is an epistemic value of a computer-verified mathematical theorem. Does its proof need to be understood by a human? (yes!)

Спасибо!