A Constructive Axiomatic Method for Physics a formal description of LIGO experiment

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[and von Neumann in 1932 for Quantum Mechanics - *A.R.*].

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While the constructive method introduces the objects of a theory only as a *genus* of things, an axiomatic theory refers to a fixed system of things [...]. There is the assumption that the domain of individuals is given as a whole [while in a constructing setting objects are built from primitive elements according to certain rules: compare Euclid's non-propositional Postulates - - *A.R.*].

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Critique

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But the examples of Newton, Clausius, von Neumann and others suggests that this form of axiomatics is inappropriate in Science (as well as in Mathematics). The fact that the century-long efforts of axiomatising GR and other fundamental physical theories did not, so far, bring commonly accepted outcomes, is another supporting evidence for that claim.

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I claim instead that the very *logical form* of the Hilbert-style axiomatic theory provides a poor representation of mathematical and scientific theories, which ignores their important *formal* features, and which is insufficient for any practical purpose (except some higher-level meta-theoretical studies).

Remedy

In what follows I describe an alternative version of Axiomatic Method motivated by Homotopy Type theory, which I call (at the absence of a better term) *constructive*, and argue that it is a better formal representation tool for physical theories.

Hilbert and Bernays 1934: a critique

The (Kantian) distinction between Form and Content is *orthogonal* to that between Hilbert&Bernays' *sharpened* axiomatics (that they call "formal"), on the one hand, and the *constructive* aka *genetic* axiomatics, which involves formal rules of building complex (non-propositional) objects from primitive objects, on the other hand.

Propositonal but Contentual

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This view of theories obviously admits a *contentual* version.

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can be read either contentfully or formally just like the corresponding *existential* axiom:

for any pair of different points there exist a straight line that joins those two points

What makes Euclid's geometry constructive?

What makes the theory based on Postulates *constructive* in the relevant sense of the word, is the logical form of Euclid's Postulates: unlike the corresponding axiom, Euclid's Postulate 1 is not a proposition that admits for a truth value but a rule applied to non-propositional objects, which should be compared with a rule of logical inference.

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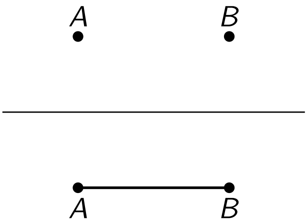
Postulate 1 inputs a pair of points and outputs a straight line.

Logical (propositional) and extra-logical (?) rules of inference

Euclid's Postulate 1 and the Introduction rule for conjunction



$$\frac{A \qquad B}{A \ \& \ B}$$

HoTT

The standard HoTT includes a set of rules (Martin-Löf Type theory with Dependent Types), which operate both at the propositional level (where they can be interpreted as standard logical rules for a constructive typed logic) and *higher* (in the sense of homotopy hierarchy) non-propositional levels.

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Higher level non-propositional objects are thought of in HoTT as *truth-makers* (aka *witnesses* aka *proofs*) of their corresponding propositions (extracted from a given homotopy type via its *propositional truncation*).

Homotopical Hierarchy of Types

Types	Logic	Sets	Homotopy
A	proposition	set	space
$a : A$	proof	element	point
$B(x)$	predicate	family of sets	fibration
$b(x) : B(x)$	conditional proof	family of elements	section
$\mathbf{0}, \mathbf{1}$	$\bot, \top$	$\emptyset, \{\emptyset\}$	$\emptyset, *$
$A + B$	$A \vee B$	disjoint union	coproduct
$A \times B$	$A \wedge B$	set of pairs	product space
$A \to B$	$A \Rightarrow B$	set of functions	function space
$\sum_{(x:A)} B(x)$	$\exists_{x:A} B(x)$	disjoint sum	total space
$\prod_{(x:A)} B(x)$	$\forall_{x:A} B(x)$	product	space of sections
Id_A	equality $=$	$\{ (x, x) \mid x \in A \}$	path space A^I

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Set does reflect non-trivial features of HoTT (including the Homotopical Hierarchy).

Rules and Axioms

MLTT is a Genzen-style (i.e., rule-based) calculus that comprises no axiom (in the usual sense of the word). The Standard HoTT adds the Axiom of Univalence (AU), which is, however got rid of in the Cubical Type theory (where AU is proved as a theorem).

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Historically, the term "axiom" was often applied to rules: Aristotle calls axiom his Perfect Syllogism, for example. Being motivated by such historical examples I take the liberty to qualify HoTT and other rules-based theories as *axiomatic*.

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Axiomatic theories in this broad sense, generally, do not justify their claims by "channelling the truth" from their axioms to those claims.

A Relevant Earlier Critique: the Semantic View

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In practical terms, Suppes' proposal is tantamount to extending Bourbaki's version of Axiomatic Method (which is not the same as Hilbert's) to science.

My Take on the Semantic View

The proponents of this view rightly stress that a typical scientific theory does not reduced to a set of true propositions. But since these people rely on the standard logical tools (Classical Predicate Logic and its Set-theoretic Semantics with some add-ons and extensions) they cannot answer the question "Where appropriate models come from?", and displace this question to the "context of discovery".

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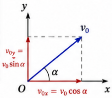
Observe, for a contrast, that even the simplest models of physical theories like the computation of trajectory of a projectile according to principles of Classical Mechanics, are built with the application of well-defined formal procedures (i.e., the appropriate mathematical *methods*) but not simply guessed and only later verified (whether they satisfy the given theory or not).

Computation of Classical trajectory

Projectile Motion: Derivation of the Trajectory

Assumptions:

- Launched from the origin (0,0)
- Initial speed v_0 at angle α above the horizontal
- No air resistance
- Constant gravity g downward



1 Equations of Motion

Horizontal motion ($a_x = 0$)

$$x(t) = v_0 \cos\alpha\, t$$

Vertical motion ($a_y = -g$)

$$y(t) = v_0 \sin\alpha\, t - \frac{1}{2} g t^2$$

These are the parametric equations of the trajectory.

2 Derive the Trajectory $y(x)$

From $x(t) = v_0 \cos\alpha\, t \quad \Rightarrow \quad t = \dfrac{x}{v_0 \cos\alpha}$

Substitute into $y(t)$:

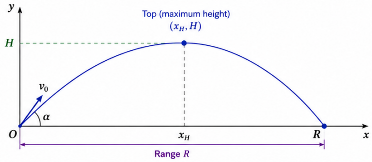
$$y = v_0 \sin\alpha \left(\frac{x}{v_0\cos\alpha}\right) - \frac{1}{2} g \left(\frac{x}{v_0 \cos\alpha}\right)^2$$

$$= x\tan\alpha - \frac{g x^2}{2 v_0^2 \cos^2\alpha}$$

Trajectory (Cartesian equation):

$$y(x) = x \tan\alpha - \frac{g}{2 v_0^2 \cos^2\alpha} x^2$$

This is a parabola opening downward.



3 Time of Flight

The projectile returns to the ground when $y(T) = 0$ (besides $t = 0$).

$$0 = v_0 \sin\alpha\, T - \frac{1}{2} g T^2$$

$$T = \frac{2 v_0 \sin\alpha}{g}$$

4 Maximum Height

At the top, vertical velocity is zero:

$$v_y(t) = v_0 \sin\alpha - gt = 0$$

$$t_H = \frac{v_0 \sin\alpha}{g}$$

Substitute into $y(t)$:

$$H = v_0 \sin\alpha \left(\frac{v_0 \sin\alpha}{g}\right) - \frac{1}{2} g \left(\frac{v_0 \sin\alpha}{g}\right)^2$$

$$H = \frac{v_0^2 \sin^2\alpha}{2g}$$

5 Horizontal Range

Range is the horizontal position when the projectile lands:

$$R = x(T) = v_0 \cos\alpha\; T = v_0 \cos\alpha \frac{2 v_0 \sin\alpha}{g}$$

$$R = \frac{v_0^2 \sin 2\alpha}{g}$$

Summary

Parametric equations:

$$x(t) = v_0 \cos\alpha\, t$$
$$y(t) = v_0 \sin\alpha\, t - \frac{1}{2} g t^2$$

Trajectory (parabola):

$$y(x) = x\tan\alpha - \frac{g}{2 v_0^2 \cos^2\alpha} x^2$$

Key results:

Time of flight: $T = \dfrac{2 v_0 \sin\alpha}{g}$

Maximum height: $H = \dfrac{v_0^2 \sin^2\alpha}{2g}$

Range: $R = \dfrac{v_0^2 \sin 2\alpha}{g}$

Theoretical methods

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Those methods make a proper part of every scientific theory deserving the name.

Newton's Apple

The above computation is a straightforward *application* of Classical Mechanics, not the discovery of its principles. Unlike the case of Newton's apple, it is not a matter of chance or stroke of genius. (But some Classical Mechanics problems can be tricky: think of the Three Body Problem and so on.)

Empirical methods

An experiment with projectile realising the above theoretical prediction, if successful (in the sense that the theoretical prediction fits the obtained empirical data), qualifies as an *material model* of the same theory.

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Notice that the truth of physical theories like Classical Mechanics is justified by observations and experiments but not by the first principles (axioms) of this theory alone.

(Neo-)Scholastic and Modern Science

The idea that a physical theory can and should be "axiomatised" in the sense that a finite number of *true* theoretical propositions called axioms would justify any other proposition of this theory via a logical (truth-preserving) inference is *wholly misleading*.

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It squares with the old-fashioned Scholastic ideal of logic-based science but it is at odds with the Modern Galileo-style science.

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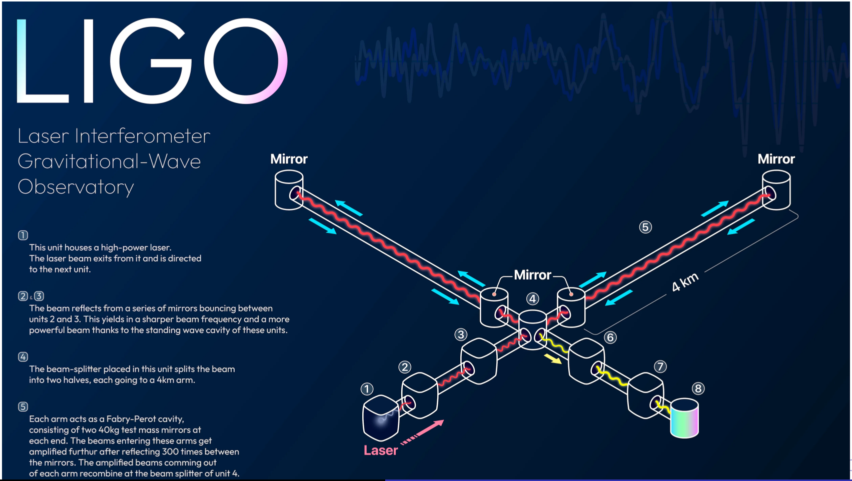
- Theoretical Semantics: Mathematical contexts (including contenual mathematical theories) in which the syntactic rules of a given theory are applied (theoretical modelling);
- Material Semantics: Empirical contexts (including experimental and observational settings) in which the syntactic rules of a given theory are applied (empirical verification of theoretical models).

LIGO


LIGO
Laser Interferometer
Gravitational-Wave
Observatory
①
This unit houses a high-power laser.
The laser beam exits from it and is directed
to the next unit.
② & ③
The beam reflects from a series of mirrors bouncing between
units 2 and 3. This yields in a sharper beam frequency and a more
powerful beam thanks to the standing wave cavity of these units.
④
The beam-splitter placed in this unit splits the beam
into two halves, each going to a 4km arm.
⑤
Each arm acts as a Fabry-Perot cavity,
consisting of two 40kg test mass mirrors at
each end. The beams entering these arms get
amplified furthur after reflecting 300 times between
the mirrors. The amplified beams comming out
of each arm recombine at the beam splitter of unit 4.
Mirror
Mirror
Mirror
4 km
Laser
①
②
③
④
⑤
⑥
⑦
⑧

Scheme of formal representation for LIGO

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Where :

- T stands for the GR formally represented in the language of HoTT and implemented in an appropriate programming language (like AGDA, Rock);
- M_T (along with arrow m) stands for a special theoretical model of GR built in an appropriate context, which in the given case is a theoretical prediction of gravitational waves (Einstein 1916). Technically speaking, it amounts to a special solution of Einstein's Field Equations (cf. the Classical case above);

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This shows that GR is not sufficient for representing LIGO: one also needs to apply further theories supporting the detection of the wanted physical phenomenon.

A formal representational scheme for LIGO

$$T \xrightarrow{m} M_T \xrightarrow{t} M_{TE} \xrightarrow{e} M_E$$

Where :

- M_E (along with arrow e) stands for an experimental design (arrow e), which brings about the material model (M_E) of T (GR), which the detectable event produced by a gravitational wave in the LIGO equipment (Abbott et al. 2016 and Castelvecchi et al. 2016).

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M_E is an experimental evidence confirming T as an *empirically adequate* theory. One can imagine the situation when M_T is built but M_E does not occur. This is what happened to Einstein's 1916 theoretical prediction before gravitational waves were first observed in 2015 with LIGO.

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- The need for such a revision is equally supported by the history of science.
- HoTT provides an alternative axiomatic architecture for physical and mathematical theories, which better squares with the epistemic goals of Modern science, and with the current scientific practice, including some novel developments such as the computer-verified mathematics.

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Köszönöm!