Euclid and Lobachevsky

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1. Key facts about Nikolai Iv. Lobachevsky (1792–1856)
2. Naturalising Geometry
3. Analysis and Synthesis
4. Conclusions

Lobachevsky's *Elements of Geometry*

Lobachevsky was a professor of mathematics (since 1814) and later a rector (in 1827–1846) in a recently founded (in 1804) provincial Russian university in Kazan. In this capacity he was supposed to produce, primarily, teaching and learning materials rather than the original research.

Research was (and still is) mostly reserved for the Russian Academy of Sciences, which also supervised teaching and the eventual research in the universities. Lobachevsky's attempts to get an approval from the Academy were not successful (the negative report by Mikhail Ostrogradsky in 1832).

Lobachevsky in 1839 as the Rector of Kazan University

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The title *Начала* used by Lobachevsky was first used for translating the title of Euclid's *Elements* by Thoma Petrushevsky in 1819 and since then became standard.

Lobachevsky adversus Euclid

Lobachevsky's attempt to revise the foundations of geometry naturally inscribes into the long-term Euclidean tradition. But Lobachevsky's revision of this tradition is very radical and it goes **far beyond** dropping the Fifth Postulate and developing a more general (non-Euclidean) geometrical theory.

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Geometry as a Natural Science

In Lobachevsky's view, Geometry is a proper part of Natural Science and Engineering. In this respect he follows d'Alembert in the *Encyclopedia* and the d'Alembert-style French geometry textbooks by Clairaut (1741), Bézou (1781), Legendre (1794), Lacroix (1795), Monge (1795).

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In the continuing debate between the defenders of Euclid in mathematics education (like Petrushevsky) and the promoters of the new d'Alembertian approach Lobachevsky definitely takes the latter side. Following in d'Alembert's steps Lobachevsky does not give his geometrical theory any axiomatic form.

Naturalising Foundations

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In addition to the foundations of Hyperbolic geometry this project included an early form of Dimension theory, which was not quite successful.

"[T]he conceptual obscurity [in the traditional foundations of Geometry] is produced by the abstractness, which is redundant in the actual measurements and hence also unnecessary in the theory itself. Surfaces, lines and points as they are [usually] defined in Geometry exist only in our imagination while the [actual] measurement of surfaces and lines is done using bodies.

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This is why it is appropriate to talk about surfaces, lines and points only as so far as these concepts are used in the actual measurement." (*The New Foundations, Intro*)

Historical error

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This myth has been designed in the early 20th century when Lobachevsky's geometrical theory was given an axiomatic form by David Hilbert, Henri Poincaré, Jean Dieudonné and others.

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A Reversal of the Epistemic Roles

Lobachevsky shares the view articulated by Condillac in his *Logic* according to which the *Analytic* geometry represents a higher level of mathematical knowledge than the traditional Euclid-style axiomatic *Synthetic* geometry.

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This is why Lobachevsky avoids using the axiomatic method quite deliberately but not just as a result of his alleged carelessness about logic.

Still a Synthesis

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But in his *Imaginary geometry* published in 1837 in French in the *Crelle Journal* Lobachevsky opts for a purely Analytic presentation of his theory.

Theoretical error

Considering Lobachevsky's achievements exclusively from an axiomatic viewpoint is not only historical but also a theoretical mistake.

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John Milnor in his 1982 paper argues that Lobachevsky's research into the Hyperbolic geometry was not properly understood before the late 1970s when William Thurston generalised some of Lobachevsky's results in his theory of Hyperbolic manifolds.

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Conclusion 1

In spite of his striking divergences from the Euclidean tradition (which are by far more significant than in Hilbert's 1899 case!) Lobachevsky, nevertheless, opts for presenting his achievements in the traditional form of the *Elements* !

Conclusion 2

Lobachevsky's project of revising the foundations of geometry in order to make this discipline more tightly connected to natural sciences remains largely open.

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Recent attempts to develop Quantum geometry and akin geometrical theories motivated by Physics are very much in Lobachevsky's spirit.

To repeat, the popular idea that the emergence of non-Euclidean geometries in the 19th century marks a detachment of Geometry from Physics and other natural sciences (defended, in particular, by Dieudonné and Stone in the 1960s) is erroneous both historically and theoretically.

Lobachevsky's Views of Geometry (2026)

https://arxiv.org/abs/2608.24941

THANK YOU!