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The Dogma of Extensionality

Russell adversus Bradley

The logic which I shell advocate is atomistic, as opposed to the monistic logic of the people who more or less follow Hegel. [Bradley] When I say that my logic is atomistic, I mean that I share the common-sense belief that there are many separate things; I do not regard the apparent multiplicity of the world as consisting merely in phases and unreal divisions of a single indivisible Reality. It results from that, that a considerable part of what one would to do to justify the sort of philosophy I wish to advocate would consist of justifying the process of *analysis*. [...]

The reason that I call my doctrine *logical* atomism is because the atoms that I wish to arrive at as the sort of last residue in analysis are logical atoms and not physical atoms.

In order to understand a propostion in which the name of a particular occurs, you must already be acquainted with that particular.

...*molecular* propositions... contain other propositions which you may call their atoms.

[excluding propositional attitudes] the truth or falsehood of the molecular proposition depends only on the truth or falsehood of the propositions that enter into it.

[extensionality; to build molecular propositions out of atomic ones - isn't this the synthesis]

... you can get down ... to ultimate simples, out of which the world is built

You find, if you read the works of physicists, that they reduce matter down to certain elements - atoms, ions, corpuscles, or what not.

(underlined by myself)

(Russell, *The philosophy of logical atomism*):

Each item can be the case or not to be the case while everything else remains the same

(Wittgenstein, *Tractatus* 1.21)

It is not true that every judgement has two ideas. We may say on the contrary that all have but one. <u>We take an ideal content, a complex totality of qualities and relations, and we then introduce divisions and distinctions, and we call these products separate ideas...</u>
[Example: wolf eating a lamb; NB:this is an <u>event</u>, not an object; <u>isn't this about analysis?</u>]

[Against <u>logical</u> atomism]
Nothing in the world that you can do to ideas, no possible torture wiil get out of them an assertion which is not universal.....
Proper names are signs of universal ....

[Style]
... idea by itself.. is an adjective divorsed, a parasite cut loose, a spirit without a body seeking rest in another, an abstraction from the concrete, a mere possibility which itself *is* nothing.
The idea is the fact with its existence disregarded, and its content mutilated. It is but a portion of the actual content cut off from its reality, and used with a reference to something else...

(Bradley, *The Principles of Logic*)

<u>Extensionalism and Intensionalism</u>

extensionalism	**intensionalism**
to analyze a thing into its elements, ultimately urelements=<u>atoms</u>, then reconstruct=synthesize it back	find (synthesize) what the thing is an element of, ultimately synthesize the Universe=<u>World</u>, then reconstruct = analyze it back
mechanism	organism
ontological atomism	ontological holism
causality (causa efficient), stuff (causa materialis)	teleonomy (causa finalis), forms (causa formalis)
gaz, particle mechanics, chemistry, etc.	solids (cristallography), biology, wave mechanics, field (ultimately general=one field) theory
liberalism: club	communitarism: tribe

Intermidiate cases: liquids, plasm (non-linear dynamics), real societies

... individual, though one and the same, has internal differences. You may hence regard it in two opposite ways. Sofar as it is one against other individuals, it is particular; sofar as it is the same throughout its diversity, it is universal.
Individaul is the identity of universal and particular.
(Bradley, *Logic*)

Set theory

By a «set» we understand every collection to a whole M of definite, well-differentiated objects m of our intuition or our thought. We call these objects the «elements» of M. (Cantor)

.. all we are interested in with sets is what members they have. ... a set is something about which we can say, of anything in the universe, either that **it** belongs to **the set** or that it does not.

types:

Level 0: individuals (urelemente)

Level 1: all collections whose members are individuals

Level 2: all collections whose members are in level 1 (or in level 0), etc.

(Drake, *Set theory*, 1974)

$x \in y =_{Def}$ set x is **element** of set y; set y is **host** of set x

Why to think of sets in terms of their elements but not in terms of their hosts?

ZF:

$x=y =_{Def} \forall z(x \in z \leftrightarrow y \in z)$ identity of indiscernibles AND indiscernibility of identicals: intensionality	$x=y =_{Def} \forall z(z \in x \leftrightarrow z \in y)$ extensionality
$\forall x \forall y(\forall z(z \in x \leftrightarrow z \in y) \rightarrow x=y)$ extensionality; (backward implication is proved)	$\forall x \forall y(x=y \rightarrow \forall z(x \in z \rightarrow y \in z))$ left substitutivity

ZF	**ZFDual**
$x=y =_{Def} \forall z(x\in z\leftrightarrow y\in z)$ intensionality	$x=y =_{Def} \forall z(z\in x\leftrightarrow z\in y)$ extensionality
$\forall x\forall y(\forall z(z\in x\leftrightarrow z\in y)\rightarrow x=y)$ **extensionality**:	$\forall x\forall y(\forall z(x\in z\leftrightarrow y\in z)\rightarrow x=y)$ **intensionality**: only one world
x is **atom** (empty set, urelement) $=_{Def}$ $\neg\exists y(y\in x)$; given extensionality the atome is unique	x is **world** (class, Absolute Reality)$=_{Def}$ $\neg\exists y(x\in y)$ given intensionality the world is unique
$\forall a\forall b(a\neq b\rightarrow\exists p\forall x(x\in p\leftrightarrow(x=a\vee x=b)))$ (ZF2) **pairing**: common area: space	$\forall a\forall b(a\neq b\rightarrow\exists p\forall x(p\in x\leftrightarrow(x=a\vee x=b)))$ (A2) **link**: common element: time
$\forall a(\exists b(b\in a)\rightarrow\exists y\forall x(x\in y \leftrightarrow\exists z\ (x\in z\ \&\ z\in a)))$ **union**: elements of elements	$\forall a(\exists b(a\in b)\rightarrow\exists y\forall x(y\in x \leftrightarrow\exists z\ (z\in x\ \&\ a\in z)))$ (A3) **intersection**: areas of areas
y is **subset** of z $=_{Def}\forall x(x\in y\rightarrow x\in z)$ every element of y is an element of z $\forall x(x\subseteq x)$	y is **superelement** of z $=_{Def}\forall x(y\in x\rightarrow z\in x)$ every area of y is an area of z $\forall x(x\supseteq x)$
$\forall a\exists y\forall x(x\in y\leftrightarrow x\subseteq a)$ **power**: set of (all the) subsets given powering no worlds (Cantor's paradox)	$\forall a\exists y\forall x(y\in x\leftrightarrow a\supseteq x)$ **root**: element of (all the) superelements given rooting no atoms
predicates generate **subsets** $\forall a\exists y\forall x\ (x\in y\leftrightarrow x\in a\ \&\ \varphi(x))$ $(x\neq x)$: <u>the</u> atom (the empty set) exists $\forall x(\varnothing\subseteq x)$	*abstractors* generate **superelements** $\forall a\exists y\forall x\ (y\in x\leftrightarrow a\in x\ \&\ \varphi(x))$ <u>the</u> world exists $\forall x(W\supseteq x)$
$\exists y(y\in x)\rightarrow\exists y(y\in x\ \&\forall z\neg(z\in x\& z\in y)$ **foundation:** types	$\exists y(x\in y)\rightarrow\exists y(x\in y\ \&\forall z\neg(x\in z\& y\in z)$ **upside down foundation**

<u>Logic</u>

5-1 Two designators **have the same extension** (in S) $=_{\text{Def}}$ they are equivalent (in S)

5-2 Two designators <u>have the same intension</u> (in S) $=_{\text{Def}}$ they are L-equivalent (in S)

2-3c A is **L-equivalent** to B (in S) $=_{\text{Def}}$ A≡B is L-true (in S)

2-2. A sentence C is **L-true** (in S) $=_{\text{Def}}$ C holds in every state-description (in S)

(Carnap *Meaning and Necessity*)

<u>Principle of extensionality:</u>

1) all functions of propositions are truth-functions, i.e. that, given anay statement which contains as a part of a proposition *p*, its true-value is unchanged if we *substitute* for p any other proposition q having the same truth -value as p;
2) in any statement containing a propositional functions any formally equivalent function may be *substituted* without changing the truth-value of the statement

(Russell*, Extensionality and Atomicity*, italic mine)

«Extensional Method of Construction»

A statement is called *extensional* if it can be transformed into an extension statement...; **otherwise**, it is called *intensional*.

The *thesis of extensionality*: there are no intensional statements; all statements are extensional

(Carnap, *Aufbau*, ch. 43-5, bold mine)

extensive (external) conjunction

a	b	a&b
t	t	t
t	f	f
f	t	f
f	f	f

extensive (external) disjunction

a	b	a∨b
t	t	t
t	f	t
f	t	t
f	f	t

intensive (internal) conjunction

()	(	&)
t	t	t
f	f	f

intensive (internal) disjunction

{}	{	∨ }
t	t	f
f	f	t

External & Internal logic with *topoi*.

Language

This charasteristic, that you can understand a proposition through the understanding its component words, is absent from the component words when those words express something simple. (Russell)

Je ne croit pas aux choses, mais aux relations entre les choses

(Jakobson citing Braque)

Family names such as *Bitter*, *Chitter*, *Ditter*, *Fitter*, *Gitter*, *Hitter*, *Jitter*, *Litter*, *Mitter*, *Pitter*, *Ritter*, *Sitter*, *Titter*, *Witter*, *Zitter*, all occur in New York. Whatever the origin of these names and their bearers, each of these vocables is used in the English of New Yorkers without colliding with their linguistic habbits. You had never heard anything about the gentelman introduced to you at a New York party. «Mr. Ditter», says your host. You try to grasp and retain this message. As an English-speaking person you, unaware of the operation, easily divide the continuous sound-flow into a definite number of successive units. Your host didn't say *bitter* or *dotter* or *ditty* but *ditter*. Thus the four sequential units capable of selective alternation with other units in English are readily educed by the listener. ...... In a message conveyed to the listener, every feature confronts him with a yes-no decision.

(Jakobson, *Fundamentals of language*)

так так так

так так так