Model-Based Knowledge and Its Representation

Andrei Rodin

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Received view

A theory is a set of propositions. It can be possibly (but not generally) represented through a list of *axioms* and described as the deductive closure of these axioms.

Non-statement view

A theory is a class of models but not an axiom system, nor its deductive closure. (Suppes, Sneed, Stegmüller, Balzer, Moulines, van Fraassen)

Suppes 2002: Term "model" is used in logic and science similarly.

One and the same theory may allow for many different axiomatizations.

Bourbaki-style representation of theories

is arguably useful for logical analysis but hardly useful for general research and educational purposes.

Model-based reasoning (AI)

Ex. (Russell&Norvig 2010):

$\text{Stroke(patient)} \rightarrow \text{Confused(patient)} \wedge \text{UnequalPupils(patient)}$

rather than

$\text{Confused(patient)} \wedge \text{UnequalPupils(patient)} \rightarrow \text{Stroke(patient)}$

(causal rather than non-causal rules; Cf. Aristotle *An. Post*)

Computational modeling techniques are highly developed and highly successful (e.g. Computational Flow Dynamics).

However making such techniques a part of digital Knowledge Representation systems remains a widely open problem.

Programs with Common Sense: McCarthy 1958

[I think of] programs to manipulate in a suitable formal language (most likely a part of the predicate calculus) common instrumental statements. The basic program will draw immediate conclusions from a list of premises. These conclusions will be either declarative or imperative sentences. When an imperative sentence is deduced the program takes a corresponding action.

Logicism vs. Anti-Logicism Debate

Anti-Logicist Objections (Bar-Hillel et al.): Logicism is too expensive and unfeasible (even if it is good in the ideal world).

"Softening" strategy: making computers more friendly to the natural language and common-sense reasoning (an impact of Analytic Philosophy?)

My view: the strategy is wrong. The real reason of failure of the standard axiomatic method is that it represents the *scientific* but not just the common-sense knowledge wrongly. Logicism in its usual form is bad for science even if it is good for the common sense!

Example: Computational Flow Dynamics

Theoretical Background: Navier-Stokes Equations:

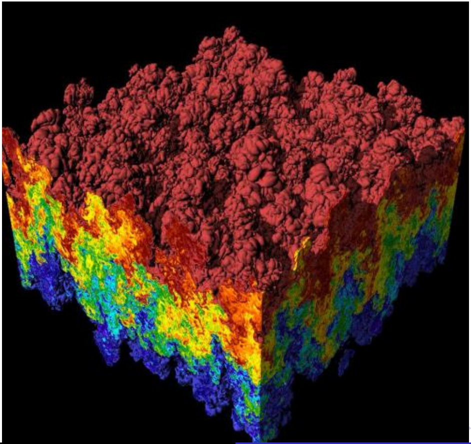
$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial[\rho u_i u_j]}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i \tag{2}$$

$$\frac{\partial(\rho e)}{\partial t} + (\rho e + p)\frac{\partial u_i}{\partial x_i} = \frac{\partial(\tau_{ij} u_j)}{\partial x_i} + \rho f_i u_i + \frac{\partial(\dot{q}_i)}{\partial x_i} + r \tag{3}$$

Computational Flow Dynamics

Computer Simulation: WATER RUNNING!



How to make these or similar computations in a standard axiomatic setting in the form of logical inferences?

some ZF axioms and modus ponens

Extensionality:

$$x = y \quad \Leftrightarrow \quad \forall z(z \in X \ \Leftrightarrow \ z \in y)$$

Pairing:

$$\exists u \forall z(z \in u \quad \Leftrightarrow \quad (z = x) \vee (z = y))$$

Union:

$$\forall u \exists v[x \in v \quad \Leftrightarrow \quad \exists w(w \in u \& x \in w)$$

• • • • •

Modus Ponens:

$$P \rightarrow Q, P \vdash Q$$

What is a formal counterpart of Navier-Stokes equations?

It is a finite but nevertheless merely *ideal* object X (like $10^{10^{10}}$) that can be studied mathematically (along with $10^{10^{10}}$). A mathematical study of X may shed some light on the Navier-Stokes and its theoretical environment.

HOWEVER X cannot help to solve or otherwise use these equations in anything like the usual sense of the word.

How X is defined?

and how one can learn its properties?

One may tell a plausible story showing that a formal reconstruction of Navier-Stokes equations is possible "in principle". X is defined as the result of this informal (sic!) procedure. The procedure is not supposed to be effective except some trivial cases. In that respect X is quite a typical *mathematical* object (compare again with $10^{10^{10}}$)

Foundations

The claim according to which X presents the Navier-Stokes and its theory in *the* standard well-founded form is a strong *philosophical* claim, which is open to philosophical objections and doubts.

Without going into a thorough discussion on foundations of mathematics one may only claim that X is a very special representation of Navier-Stokes, which proves useful for logical analysis (of a very particular sort) but certainly NOT for a all-purpose representation of this theory and the associated mathematical knowledge.

Is the controversy btw. the received and the non-statement views on theories reconcilable?

Hilbert 1899 (but not after Hilbert&Ackermann 1928), Hintikka 2011: YES!

The main purpose of axiomatization is fixing a class of models and organaizing its structure by logical means. The "right" notion of *logical inference* is that of *semantic* consequence $\models$ rather than that of *syntactic* consequence (aka formal derivation) $\vdash$. The only purpose of the latter is to reflect the former.

$A_1, A_2, \ldots, A_n \models B$ iff every interpretation that satisfies $A_1, A_2, \ldots, A_n$ also satisfies B (i.e., B is "true in every model").

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- The above notion of semantic consequence is designed to provide a universal concept of logical inference applicable to any domain. However it is not obvious that such a concept makes good sense. There may exist <u>no</u> literally "universal" domain, which may provide this notion of inference with any definite content.

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- A theory T is a class of models and
- T can be fully determined by its axioms (even if a finite axiomatic representation is not always possible, if it is possible it is not unique, and not all logical consequences of the axioms can be known through formal derivations).

Problem 1

In a standard (formal) axiomatic theory axioms provide a propositional *description* of its models, i.e., they provide a necessary and sufficient *condition* for being a model of this given theory. However the axioms give no clue as to *how* the class (or at least some) of such models can be found or built.

Where the Models Come From?

Hilbert&Hintikka: The models are found or built by empirical trials and errors, intuitive guesses etc. while the corresponding axiomatic description of these models is a way of putting these pieces of knowledge (i.e., the informal fragmentary models) into a logical order.

The question belongs not to Logic but to Psychology of mathematical and scientific discovery.

Problem 2

It is not evident that the above answer gives the full justice to the successful scientific practice. Apparently it ignores the fact that successful scientific theories (e.g. Classical Mechanics or Flow Dynamics) allow for a routine *generation*of specific models from some basic elementary pieces by following certain simple rules.

Hilbert 1900 and Hilbert&Bernays 1936: *genetic* aka *constructive* method.

Genetic Method

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- Mathematics (Hilbert 1900): Dedekind Cuts. Notice that Dedekind's procedure is not constructive in anything like the usual sense of the term.

Question

Does the standard axiomatic method fully cover the genetic method? HoTT suggests an answer in negative.

CS rule of thumb

Curry-Howard Correspondence: Propositions-as-Types

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- Propositional identity of terms x, y of (definitionally) the same type A:
  $Id_A(x, y) : type$;
  Remark: propositional identity is a (dependent) type on its own.

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- Extensionality: Propositional identity implies definitional identity (ex. LCCC)

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- First intensional (albeit 1-extensional) model: Hofmann & Streicher 1994:
  groupoids instead of sets
  families groupoids indexed by groupoids instead of families of sets indexed by sets

Hofmann & Streicher groupoid model

$\vdash A : type$ - groupoid A
$\vdash x : A)$ - object x of groupoid A
$Id_A(x, y) : type$ - arrow groupoid $[I, A]_{x,y}$ of groupoid A
(no reason to be trivial unless x = y!)

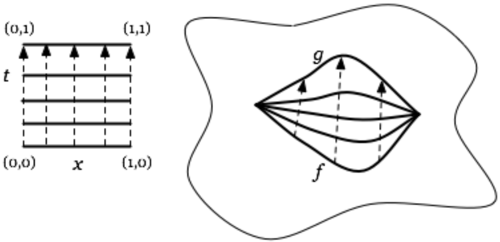
MLTT: Higher Identity Types

- $x', y' : Id_A(x, y)$
- $Id_{Id_A}(x', y') : type$
- and so on

HoTT: the idea

Types in MLTT can be modeled by spaces (up to homotopy equivalence) in Homotopy theory, or equivalently, by higher-dimensional groupoids in category theory. (Voevodsky circa 2008).

Path Homotopy



Homotopy model of MLTT

- In the groupoid model of MLTT groupoids are *fundamental groupoids* (i.e., groupoids of paths) of topological spaces .
- Higher (homotopical) groupoids model higher identity types. Intensionality all way up (Voevodsky circa 2008).

Propositions-as-**Some**-Types !

Which types are propositions?

Def.: Type P is a *mere proposition* if $x, y : P$ implies $x = y$ (definitionally).

Propositional reduction as truncation

Each type is "made into" a (mere) proposition when one ceases to distinguish between its terms, i.e., *truncates* its higher-order homotopical structure.

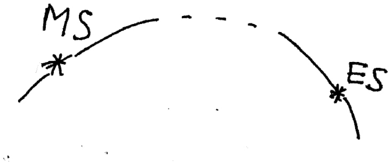
Interpretation: Truncation reduces the higher-order structure to a single element, which is **truth-value**: for any non-empty type this value is **true** and for an empty type it is **false**.
The reduced structure is the structure of **proofs** of the corresponding proposition.
To treat a type as a proposition is to ask whether or not this type is instantiated without asking for more.

- Thus in HoTT "merely logical" rules (i.e. rules for handling propositions) are instances of more general formal rules, which equally apply to non-propositional types.
- These general rules work as rules of building models of the given theory from certain basic elements which interpret primitive terms (= basic types) of this given theory.

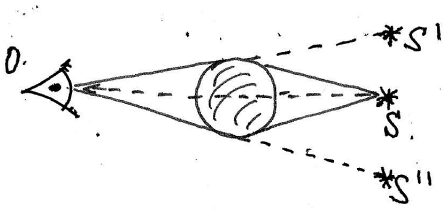
Thus HoTT provides a precise sense in which a theory may not reduce to a set of its propositions. In addition to the propositional *knowledge what* HoTT represents a non-propositional *knowledge how* for non-propositional types.

Notice that the standard axiomatic presentation also involves a presentation of *knowledge how* in the form of logical rules applicable to propositions. In HoTT corresponding rules are applicable also to non-propositional types.

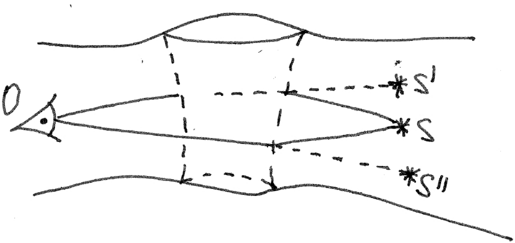
Identity through time:



Gravitational lensing

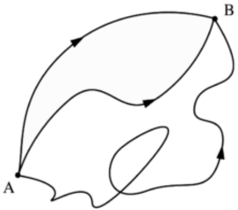


Wormhole lensing



Quantum trajectories

(truncation: quantum $\longrightarrow$ classical)



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- HoTT supports and formalizes a concept of reasoning, which does not reduce to inferring propositions from some other propositions but also involves constructing of non-propositional objects (which serve for proving propositions).
- For this reason the successful formalization of mathematics with HoTT (Univalent Foundations) has better chances to be useful in Model-Based Knowledge Representation than the standard axiomatic architecture, which is used for this purpose at present.

THE END