Justification Procedures in Knowledge Representation

Constructive Knowledge - 4 (IP RAS, November 15, 2017)

Epistemological Generalities and some Concerns about the Justification of Knowledge in KR systems

McCarthy and Hayes on Philosophical Foundations of AI and KR

Two Alternative Semantics of Logical Inference

Univalent Foundations as a New Paradigm for KR

Conclusion

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- **Belief** is subjective: it is a mental state of an individual epistemic agent (usually expressed linguistically).
- **Justification** (aka **Proof**): combines subjective and objective features. It (i) generates a belief and (ii) is a subject of public challenges and validity checks. A procedure that generates a true belief B in a given epistemic agent does not necessarily qualify as a justification of B.

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- It does not provide users with explicit evidences (in the support) of truths (= true propositions) conveyed by KR systems; the task justification of represented knowledge is divided between the suppliers of raw data and computer engineers.
- This endangers the public character of scientific knowledge and increases the role of epistemic authority.
- It makes the represented knowledge less reliable.

Desideratum for KR systems

Provide individual users with evidences justifying propositions qualified by the given KR system as part of public or corporative knowledge. In case of empirical justification this may include (i) certified access to raw data, (ii) guarantees that the raw data have been obtained from appropriate sources and remain well-preserved ever since.

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Claims

- The neglect of effective user-friendly justification procedures in the existing KR systems (is not a merely technical issue but) is conditioned by the logical architecture of such systems, which in its turn implements the mainstream conception of logic (and, more specifically, the semantics of logical inference) developed in the 20th century.
- An alternative conception of logic developed within the constructive/intuitionistic tradition helps to solve this problem via a revision of the basic logical architecture of KR systems. Moreover the alternative constructive architecture turns out to be more computer-friendly than the standard architecture: it allows one to bridge a gap between proofs and computations that is present in the standard case.

McCarthy and Hayes 1969

The right way to think about the general problems of metaphysics and epistemology is not to attempt to clear one's own mind of all knowledge and start with 'Cogito ergo sum' and build up from there. Instead, we propose to use all of our knowledge to construct <u>a computer program that knows</u>. [..]

Remark

The authors intend to attribute knowledge to a computer program rather that make this this program into an instrument of human knowledge.

McCarthy and Hayes 1969

This point of view corresponds to the presently dominant attitude towards the foundations of mathematics. We study the structure of mathematical systems from the outside as it were using whatever metamathematical tools seem useful instead of assuming as little as possible and building up axiom by axiom and rule by rule within a system.

Remark

Apparently the authors tend to identify knowledge with a meta-theory of action.

McCarthy and Hayes 1969

[W]e want a computer program that decides what to do by inferring in a formal language that a certain strategy will achieve its assigned goal. . . . A method is given of constructing a sentence of first order logic which will be true in all models of certain axioms if and only if a certain strategy will achieve a certain goal.

Remark

Although the authors express their liberal attitude towards possible meta-mathematical tools for KR as a matter of fact they propose to use for this purpose the first-order logic, which is used in the contemporary mainstream foundations of mathematics.

Ironically, this particular formal tool is designed as an implementation of conception of logic, according to which logic is an ontological rather than an epistemic tool.

Logical consequence according to Tarski

Propositional form B is a logical consequence of propositional forms $A_1, \ldots, A_n$ iff every interpretation I of the given language, which makes $A_1, \ldots, A_n$ into true propositions $A_1^I, \ldots, A_n^I$ makes B into true proposition B^I, in symbols $A_1, \ldots, A_n \models B$.

Tarski's conception of consequence explains *rules* of inference like $A_1, \ldots, A_n \models B$ via a meta-theoretical *propositions*. The resulting "Platonic" conception of logic does not qualify the notion of rule as fundamental.

The notion of rule from this point of view is syntactic: syntactic deductions of form $A_1, \ldots, A_n \vdash B$ symbolically reflect *facts* of the matter that certain relations of logical consequence hold.

The resulting conception of logic does not have any epistemological import: logical inferences merely reflect syntactically the relations (of logical consequence), which hold between certain things in this world independently of one's epistemic attitude if any.

I doubt that a system of logic construed in this way can be made into an epistemic tool by providing it with some additional features such as epistemic modalities.

Gentzen: Rules versus Axioms

“The formalization of logical deduction, especially as it has been developed by Frege, Russell, and Hilbert, is rather far removed from the forms of deduction used in practice in mathematical proofs. ”(Gentzen 1935)

Natural Deduction and Sequent Calculus

Gentzen 1935: semantics via introduction rules

The introductions [i.e. introduction rules] represent, as it were, the 'definitions' of the symbol concerned.

Implicit Definitions

Compare Gentzen's idea to use syntactic rules as a form of implicit definitions with Hilbert's use of axioms as implicit definitions. The two approaches may appear to be very similar but in fact they are not.

Think of usual axioms of Group Theory. These axioms serve as a definition in the following sense: any structure, which satisfy the axioms, i.e., serves as their model, is a group. Model theory, which originates from Tarski's pioneering works, explains away the *satisfaction relation* in terms of truth-conditions.

Implicit Definitions (continued)

What kind of entity X can possibly “satisfy” a *rule* or a system of rules, so one could claim that the rules “define” X in some reasonable sense? How the satisfaction relation (if it can be used here at all) has to be construed in this case?

Prawitz: General Proof Theory

In model theory, one concentrates on questions like what sentences are logically valid and what sentences follow logically from other sentences. But one disregards questions concerning how we *know* that a sentence is logically valid or follows logically from another sentence. General proof theory would thus be an attempt to supplement model theory by studying also the evidence or the process - i.e., in other words, the proofs - by which we come to know logical validities and logical consequences. (Prawitz 1974)

Martin-Löf 1985: Meaning Explanation

is analogous to program compiler: translation of the given syntax into elementary self-evident steps of reasoning ()

Proof-theoretic Semantics

PTS is not denotational. It does not assign certain entities to certain symbols. It assigns to symbols (and first of all to logical symbols, i.e., to logical constants) their *meaning*, which is not construed in this case as an entity. The procedure of such an assignment is called after Martin-Löf 1985 the *meaning explanation* and consists, roughly, of the explication of computational content of logical constructions in terms of their building blocks, which are presented in a self-explanatory canonical form.

A Terminological Issue

Is it justified to call the Gentzen-style formal approach *axiomatic* even if it is rule-based rather than axiom-based?

I tend to answer in positive having in mind Aristotle's use of the term "axiom". Axiomatic method is more general than we learn it from Hilbert.

Are Hilbert- and Gentzen-style axiomatic presentations inter-translatable?

Only limitedly and only at the syntactic level. Since an axiom is a rule with the empty set of premises the Gentzen-style is more basic. Under certain strong conditions a sequent calculus is deductively equivalent to a Hilbert-style theory with the only rule of modus ponens (Krupski).

However there is no regular procedure known to me that supports a *semantic* translation between the two axiomatic styles. In particular, it is unclear how extra-logical rules in theories can be systematically dispensed with.

Gentzen-style beyond Logic?

Until recently the Gentzen-style rule-based formal approach has been used only in purely logical calculi (with the only exception of formal arithmetic). Unlike Hilbert Gentzen never attempted to use his version of axiomatic method in all areas of mathematics or in sciences. Univalent Foundations is the first attempt to use this approach more widely in mathematics. I suggest that it can be also effectively used in Knowledge Representation in general.

Advantages of Gentzen-style formal presentations

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- Allow for managing proofs of very different kinds, which include both mathematical arguments and empirical evidences;
- Support the representation of all sorts of *methods* including extra-logical mathematical methods, experimental methods, etc.;
- Are computer-friendly, i.e., allow for encoding into suitable program languages.

MLTT: Syntax

- 4 basic forms of judgement:
  (i) $A : TYPE$;
  (ii) $A \equiv_{TYPE} B$;
  (iii) $a : A$;
  (iv) $a \equiv_A a'$
- Context : $\Gamma \vdash$ judgement (of one of the above forms)
- no axioms (!)
- rules for contextual judgements; Ex.: dependent product :
  If $\Gamma, x : X \vdash A(x) : TYPE$, then $\Gamma \vdash (\Pi x : X)A(x) : TYPE$

MLTT: Semantics of $t : T$ (Martin-Löf 1983)

- t is an element of set T
- t is a proof (construction) of proposition T ("propositions-as-types")
- t is a method of fulfilling (realizing) the intention (expectation) T
- t is a method of solving the problem (doing the task) T (BHK-style semantics)

Sets and Propositions Are the Same

If we take seriously the idea that a proposition is defined by lying down how its canonical proofs are formed [. . .] and accept that a set is defined by prescribing how its canonical elements are formed, then it is clear that it would only lead to an unnecessary duplication to keep the notions of proposition and set [. . .] apart. Instead we simply identify them, that is, treat them as one and the same notion. (Martin-Löf 1983)

MLTT: Definitional aka judgmental equality/identity

$x, y : A$ (in words: x, y are of type A)

$x \equiv_A y$ (in words: x is y by definition)

MLTT: Propositional equality/identity

$p : x =_A y$ (in words: x, y are (propositionally) equal as this is evidenced by proof p)

Definitional eq. entails Propositional eq.

$$\frac{x \equiv_A y}{p : x =_A y}$$

where $p \equiv_{x=_A y} refl_x$ is built canonically

Equality Reflection Rule (ER)

$$\frac{p : x =_A y}{x \equiv_A y}$$

ER is not a theorem in the (intensional) MLTT (Streicher 1993).

Extension and Intension in MLTT

- MLTT + ER is called *extensional* MLTT
- MLTT w/out ER is called *intensional*
  (notice that according to this definition intensionality is a negative property!)

Higher Identity Types

- $x', y' : x =_A y$
- $x'', y'' : x' =_{x =_A y} y'$
- $\ldots$

HoTT: the Idea

Types in MLTT are (informally!) modeled by spaces (up to homotopy equivalence) in Homotopy theory, or equivalently, by higher-dimensional groupoids in Category theory (in which case one thinks of *n*-groupoids as higher homotopy groupoids of an appropriate topological space).

Homotopical interpretation of Intensional MLTT

- $x, y : A$
  x, y are points in space A
- $x', y' : x =_A y$
  x', y' are paths between points x, y; $x =_A y$ is the space of all such paths
- $x'', y'' : x' =_{x =_A y} y'$
  x'', y'' are homotopies between paths x', y'; $x' =_{x =_A y} y'$ is the space of all such homotopies
- ...

Point

Definition

Space S is called contractible or space of h-level (-2) when there is point p : S connected by a path with each point x : A in such a way that all these paths are homotopic (i.e., there exists a homotopy between any two such paths).

Homotopy Levels

Definition

We say that S *is a space of h-level* $n+1$ *if for all its points* x, y *path spaces* $x =_S y$ *are of h-level* n*.*

Cummulative Hierarchy of Homotopy Types

- -2-type: single point *pt*;
- -1-type: the empty space ∅ and the point *pt*: truth-values aka (mere) propositions
- 0-type: sets: points in space with no (non-trivial) paths
- 1-type: flat groupoids: points and paths in space with no (non-trivial) homotopies
- 2-type: 2-groupoids: points and paths and homotopies of paths in space with no (non-trivial) 2-homotopies
- ...

Propositions-as-**Some**-Types !

Which types are propositions?

Def.: Type P is a *mere proposition* if $x, y : P$ implies $x = y$ (definitionally).

Truncation

Each type is transformed into a (mere) proposition when one ceases to distinguish between its terms, i.e., *truncates* its higher-order homotopical structure.

Interpretation: Truncation reduces the higher-order structure to a single element, which is **truth-value**: for any non-empty type this value is **true** and for an empty type it is **false**.
The reduced structure is the structure of **proofs** of the corresponding proposition.
To treat a type as a proposition is to ask whether or not this type is instantiated without asking for more.

- Thus in HoTT "merely logical" rules (i.e. rules for handling propositions) are instances of more general formal rules, which equally apply to non-propositional types.
- These general rules work as rules of building models of the given theory from certain basic elements which interpret primitive terms (= basic types) of this given theory.
- Thus HoTT qualify as *constructive* theory in the sense that besides of propositions it comprises non-propositional objects (on equal footing with propositions rather than "packed into" propositions as usual!) and formal rules for managing such objects (in particular, for constructing new objects from given ones). In fact, HoTT comprises rules with apply *both* to propositional and non-propositional types.

Conclusion

McCarthy and Hayes were quite right when they looked at their contemporary foundations of mathematics as a theoretical basis for KR. But following in their footsteps today amounts to looking at the Univalent Foundations as a possible basis of KR architecture rather than continuing to develop their original approach. This is more than just a new mathematical fashion because it addresses the open problem of justification outlined in the beginning of this talk.

THANK YOU