

Directed Homotopy Type Theory and the (In)vertibility of Mathematics

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24 апреля 2019 г.

Knowledge in the objective sense consists not of thought processes but of thought *contents*. It consists of the content of our linguistically formulated theories; of that content which can be, at least approximately, translated from one language into another . The objective thought content is that which remains invariant in a reasonably good translation. Or more realistically put: the objective thought content is what the translator tries to keep invariant, even though he may at times find this task impossibly difficult. (Three Worlds, Tanner Lecture on Human Values delivered at the University of Michigan on April 7, 1978; italic is Popper's, underlining mine)

[A]n objective fact is one that is invariant under all admissible transformations. [. . .] We also have learned (aposteriori) that the objective transformations marking an objective fact (as described by our fundamental theory) form a mathematical group.
(Invariances: the Structure of the Objective World, 2001, p. 82)

Noether Theorem

If a system has a continuous symmetry property, then there are corresponding quantities whose values are conserved in time.

All infinite cyclic groups are isomorphic, but this infinite group appears over and over again - in number theory, in ornaments, in crystallography, and in physics. Thus, the "existence" of this group is really a many-splendored matter. An ontological analysis of things simply called "mathematical objects" is likely to miss the real point of mathematical existence. (Structure in Mathematics, PM 1996)

The subject matter of pure mathematics is invariant form, not a universe of mathematical objects consisting of logical atoms.
(Structure in Mathematics and Logic: A categorical Perspective, PM 1996)

My alternative slogan

The subject matter of pure mathematics is functorial transformation, not invariant form. (Categories Without Structures, PM 2011)

Pythagoras Theorem

▶ Euclid Elements 1.47:

In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.

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- ▶ Alfred Doneddu 1965:

If two non-zero vectors x and y are orthogonal then

$$(y - x)^2 = y^2 + x^2$$

[W]e may have in the bulk term [e.g. water] a relic, half vestigial and half adapted, of a pre-individuative phase in the evolution of our conceptual scheme. And some day, correspondingly, something of our present individuative talk may in turn end up, half vestigial and half adapted, within a new and as yet unimagined pattern beyond individuation. Transition to some such radically new pattern could occur either through a conscious philosophical enterprise or by slow and unreasoned development along lines of least resistance. A combination of both factors is likeliest. (Ontological Relativity and Other Essays, 1969 p. 24)