

DTT (after Warren 2013)

- 1) MLTT rules
- 2) 1 base type (A)
- 3) directed types aka hom-types

Formation: $(\Gamma) \vdash A : \text{TYPE}$

$x, y : A \vdash A(x, y)$

Intro:

$$\frac{\vdash A}{x : A \vdash \lambda(x) : A(x, x)}$$

↖ Reflection

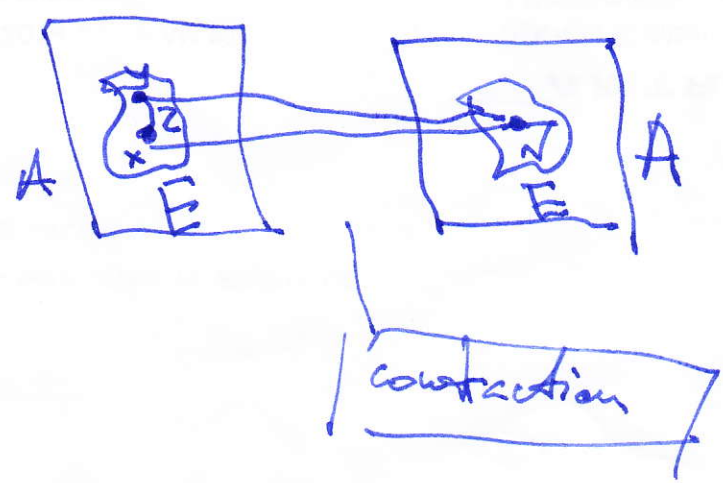
Elim

for Ids

$x, y : A, z : \text{Id}_A(x, y) \vdash E : \text{TYPE}$

$x : A \vdash e : E[x/y, \text{ref}/z]$

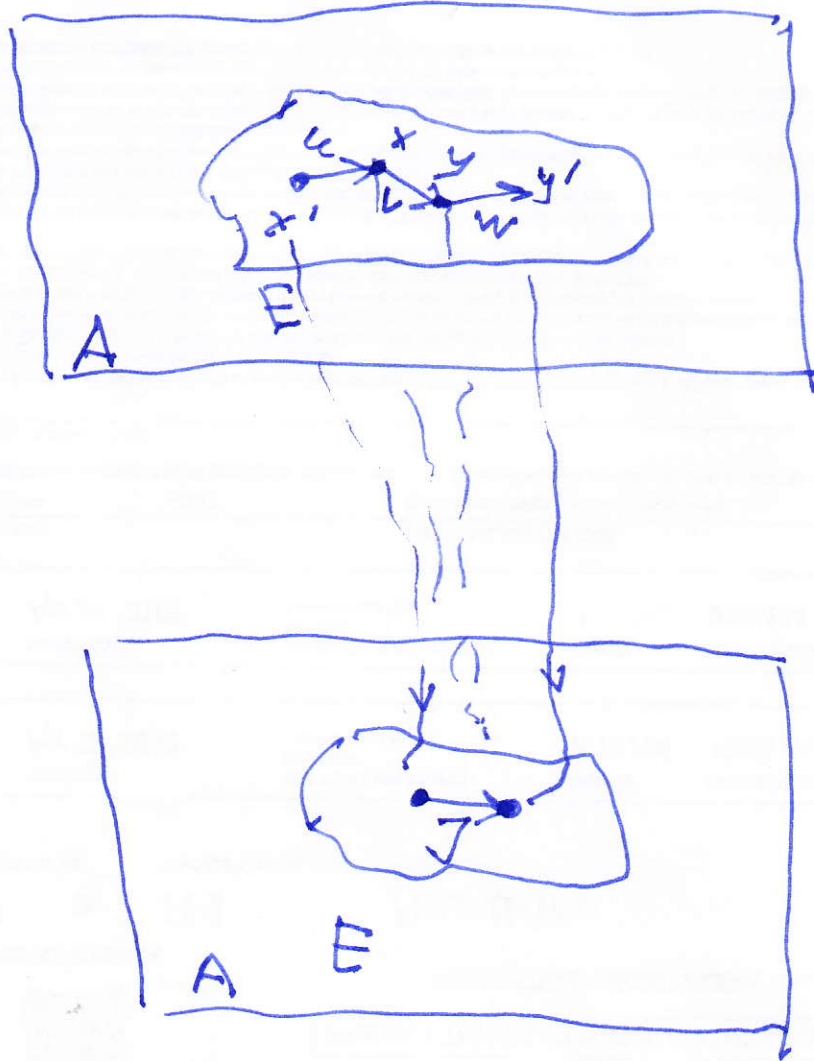
$x, y : A, z : \text{Id}_A(x, y) \vdash J(e, x, y, z) : E$



(Indiscernible of Identities)

Elim for hom

(2)



$$\begin{array}{l}
 x', x, y, y' : A, \\
 u : A(x, x) \\
 v : A(x, y) \\
 w : A(y, x')
 \end{array}
 , \Delta \vdash E : \text{TYPE}$$

$$\frac{x, y : A \quad v : A(x, y)}{\vdash e : E[x/x', r(x)/u, y/y', r(y)/w]}$$

$$\frac{x', x, y, y' : A \quad \begin{array}{l} u : A(x', x) \\ v : A(x, y) \\ w : A(y, y') \end{array}}{\vdash \lambda (a, x', x, y, y', u, v, w) : E}$$

Not valid in MLTT unless...

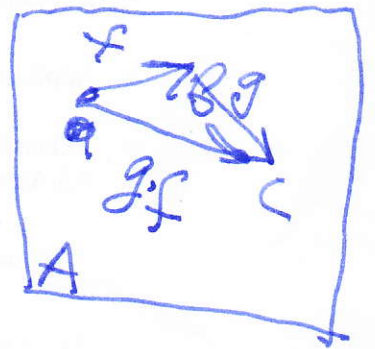
3
 The following 2 conditions are not

(c1) If one of $\{x', x, y, y'\}$ are free in Δ , then also in E

(c2) x, y are free in $\left[\frac{x}{x}, \frac{y}{y}, \frac{r(y)}{w} \right]$

Composition

$f: A(a, b), g: A(b, c)$



$$g \circ f = \lambda(e, f, g, r(c))$$

$$(2) f = \lambda(e, r(a), f, g)$$