INTRO TO MATHEMATICS
FALL 2021

ANDREI RODIN
(ANDREI@PHILOMATICA.ORG)

Course Description:
This introductory course covers issues related to the axiomatic architecture and conceptual foundations of modern mathematics including elements of Set theory, Group theory, Category theory and Topology. In this course mathematics is treated as a human endeavour, an experimental site for human thought, which has a large impact on other disciplines including natural sciences and humanities.

When: Fridays 16h40 —20h

Where: Zoom Meeting

`https://us02web.zoom.us/j/81646474370?pwd=eksrdng3dkVSeHBUYWtNdytZR1JTUT09`

Meeting ID: 816 4647 4370
Passcode: 645174

Office Hour: Fridays 15h15 —16h15, by appointment

Schedule (by weeks):

Class 1: Mathematics and the Real World. Mathematics in Society. Mathematics as a Liberal Art. Mathematics as a Science. Foundations of Mathematics. Conceptual and Logical Foundations. A Brief History of Foundations: Numbers, Figures, Magnitudes, Sets, Categories. Problems, Theorems, Proofs and Constructions.

Class 2: Basic Set-theoretic notation. Extensionality and Separation. Boolean operations on sets. Abstract algebra.

Class 3: Propositional Logic. Logical Connectives and Truth Tables. Boolean Algebra of Propositions. Exercises

Test 1 (Language of sets and Boolean Algebra)

Class 4: Combinatorics: permutations, combinations, arrangements.

Class 5: Principle of Mathematical Induction. Proofs by Induction. Peano Arithmetics. Exercises.

Class 6: Numeration Systems.

Test 2 (numeration systems, combinatorics and induction)

Class 7: Axiomatic and genetic introduction of number concepts. Whole, Rational, Real and Complex Numbers. Applications of Real and Complex numbers.

Class 8: Infinite Sets. One-to-one correspondence between sets. Cardinality of sets. Cantor Theorem and Continuum Hypothesis. Countable and non-countable sets.

Class 9: Cartesian Product of sets. Relations defined on sets. Equivalence and Partial Order.

Class 10: Functions as maps. Composition of functions. Invertible and non-invertible functions. Injective and surjective maps.

Test 3 (complex numbers, cardinality, relations and functions)

Class 11: Algebraic Groups. Groups of permutations. Subgroups. Groups of geometrical transformations. Groups of Symmetry in Geometry. Example: regular polyhedra. Lagrange theorem and Cayley theorem. Applications of Group theory.

Class 12:

Group Homomorphisms. Mathematical structures. Language of Categories. Morphisms and Isomorphisms. Groupoids. Categories of Sets, Groups, Graphs, Topological Spaces. Groups and Partial orders as categories. Initial and terminal objects. Elements of sets as morphisms.

Class 13: Topological spaces and Metric spaces. Konigsberg Bridges and Euler Characteristics of polyhedra. Applications of topology in Data Analysis.

Class 14: Homotopy. Fundamental groups and groupoids of topological spaces. Brouwer's Fix Point theorem.

Test 4 (Group theory, Category theory and Topology)

Class 15 Review and Preparation for the Final Exam.

The final mark comprises:

- work in the classroom, participation in discussions: 20 percents

- intermediate tests: 30 percents

- final exam: 50 percents