

PROOFS AND FUNDAMENTALS SPRING 2022

ANDREI RODIN
(ANDREI@PHILOMATICA.ORG)

Professor: Andrei Rodin (andrei@philomatica.org , a.rodin@spbu.ru; web: philomatica.org)

When: Mondays 16h20-20h (Moscow time) from Feb14 through May 23

Where: Zoom (so far): Meeting ID: 851 2378 2750; Passcode: 470566

Office Hours: Mondays 15h-16h Zoom by appointment

Course Description:

This course covers a basic theory of mathematical proofs and introduces elements of computer-assisted proofs.

Class 1: Knowledge and Belief. Proof and Justification. Proofs in Science, Court, and Mathematics. Examples of mathematical proofs: Angle Sum theorem, Non-Commensurability of a square side with its diagonal, Infinity of Primes (reductio ad absurdum)

Class 2: Logical Form and Non-Logical Content. Is mathematical reasoning logical? Analytic and Synthetic judgements in mathematics. Constructive and non-Constructive reasoning in mathematics.

Class 3: Classical propositional calculus. Tautologies and contradictions. Syntax and Semantics. Truth and Logical Validity. Hilbert- and Gentzen-style formal calculi.

Class 4: Propositional Logic: Natural Deduction

Class 5: Propositional Logic: Semantics

Reading: Avigad, Section 6

Test 1: Propositional Logic

Class 6: First-Order Logic: the Idea (Predicates and Quantifiers)

Class 7: Classical First-Order Logic : Natural Deduction

Class 8: Theories and Models. Truth in a Model

Class 9: Logical Consequence. Soundness and Completeness of Classical First-Order Logic

Reading: Avigad, Section 10, part 2 (with exercises)

Test 2: First-Order Logic

Class 10: Model-Theoretic and Proof-Theoretic Logical Semantics

Class 11: Axiomatic Theories. Semantic View of Theories. Formal Semantics.

Class 12: Constructive and Non-Constructive Reasoning Revisited. Intuitionistic Propositional Logic and its BHK-semantics.

Test 3:

Class 13: Proofs and Computations. Lambda-Calculus and Curry-Howard Correspondence.

Class 14: Computer-Assisted Proofs

Test 3: Lambda-Calculus and Constructive Reasoning

Class 15: Formal Proofs and Human Understanding in Mathematics and Beyond

Classes 16 Exercises and Preparation for the Exam.

The final mark comprises:

- work in the classroom, participation in discussions: 20 percents
- 3 written intermediate tests: 40 percents
- final exam: 40 percents

The exam involves 2 written exercises and a short oral interview during which the examinee is asked to explain in few words her solution of the exercise.

Literature (to be extended):

J. Avigad et al., https://leanprover.github.io/logic_and_proof/index.html

E.D. Bloch, *Proofs and Fundamentals*, Springer 2000

A. Rodin, On Constructive Axiomatic Method, *Logique et Analyse*, vol. 61, no 242 (2018), p. 201-231

A. Rodin, Two Styles of Axiomatization: Rules versus Axioms. A Modern Perspective. *Bulletin of Symbolic Logic*, vol. 24, n. 2 (2018), p. 263-264